Rock failure
Types of tests on rocks
Hydrostatic compression

\[ S_0 = S_1 = S_2 = S_3 \]

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Uniaxial compression

\[ S_0 \neq 0 \quad S_2 = S_3 = 0 \]

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Triaxial compression

\[ S_1 > S_2 = S_3 \]

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Triaxial extension

\[ S_1 = S_2 > S_3 \]

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True triaxial

\[ S_1 \neq S_2 \neq S_3 \]

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Mohr's circles

\[ \tau_f = \frac{1}{2} (\sigma_1 - \sigma_3) \sin(2\beta) \]

\[ \sigma_n = \frac{1}{2} (\sigma + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos(2\beta) \]
Mohr Envelope

Failure occurs when: $\tau = f(\sigma_n)$

$\tau$ = Shear stress
$\sigma_n$ = Normal stress

$\sigma_1$ = UCS ($C_0$)

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Linearized Mohr Envelope

$$\tau$$

$$\sigma_1 = 0$$  $$\sigma_3$$  $$\sigma_1 = \text{UCS} \ (C_0)$$

$$\mu_i \ (\text{coefficient of internal friction})$$

$$\beta$$

$$_{\sigma_1}$$  $$\sigma_3$$

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Mohr-Coulomb failure

\[ \tau = S_0 + \sigma_n \mu_i \]

\[ C_0 = 2S_0 \left( \sqrt{\mu_i^2 + 1} + \mu_i \right) \]
Triaxial tests on sandstone

\[ \mu_i = \frac{n - 1}{2\sqrt{n}} \]

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Mohr Envelope for Sandstone

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Cohesion and internal friction data

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Cohesion and internal friction data

![Diagram showing cohesion and internal friction data](image)

- **Strong Rock** (higher cohesion)
- **Weak Rock** (lower cohesion)

Parameters:
- $\sigma_r = \sigma_3$
- $\sigma_\theta = \sigma_1$
- $P_m = P_0$
- $P_m > P_0$

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Yield surface

Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia
\( \pi \)-plane

Mohr Coulomb Yield Surface 3Db. Licensed under CC BY-SA 3.0 via Wikipedia
Pressure dependence

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Other failure criteria

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Recall: Mohr Envelope for Sandstone

![Mohr Envelope Diagram](image)

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Hoek-Brown criterion (parabolic fitting)

\[ \sigma_1 = \sigma_3 + C_0 \sqrt{\frac{m \sigma_3}{C_0} + s} \]

\( m \) and \( s \) are fitting parameters that depend on rock properties and the degree of fracturing.

Typical values

<table>
<thead>
<tr>
<th>Typical Range of ( m )</th>
<th>Types of rocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 &lt; m &lt; 8 )</td>
<td>carbonate rocks (dolomite, limestone, marble)</td>
</tr>
<tr>
<td>( 4 &lt; m &lt; 10 )</td>
<td>lithified argillaceous rocks (sandstones, quartzite)</td>
</tr>
<tr>
<td>( 15 &lt; m &lt; 24 )</td>
<td>arenaceous rocks (andesite, dolerite, diabase, rhyolite)</td>
</tr>
<tr>
<td>( 22 &lt; m &lt; 33 )</td>
<td>course-grained polyminerallc gneous and metamorphic (amphibolite, gabbro, gneiss, norite, quartz-diorite)</td>
</tr>
</tbody>
</table>

Intact Rocks -- \( s \rightarrow 1 \)

Completely Granualated -- \( s \rightarrow 0 \)
Lade Criterion

\[
\left( \frac{I_1^3}{I_3} - 27 \right) \left( \frac{I_1}{p_a} \right)^{m'} = \eta_1
\]

with

\[
I_1 = S_{ii} = S_1 + S_2 + S_3 \text{ (first invariant of } S) \]

\[
I_3 = \det(S) = S_1 S_2 S_3 \text{ (third invariant of } S) \]

\[p_a\] is atmospheric pressure, \(m'\) and \(n_1\) are material constants
Modified Lade Criterion (dependence on $\sigma_2$)

\[
\begin{pmatrix}
(\sigma_1')^3 \\
I_3'
\end{pmatrix} = 27 + \eta
\]

with

\[I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S)\]
\[I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S)\]
\[S = \frac{S_0}{\tan \phi}\]
\[\eta = \frac{4(\tan \phi)^2(9 - 7 \sin \phi)}{1 - \sin \phi}\]
\[\tan \phi = \mu; \text{ and } S_0 \text{ from Mohr-Coulomb criterion}\]
Comparison

Mohr-Coulomb

modified Lade

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Others

- modified Wiebols-Cook
- Druker-Prager
- many more!
Strength anisotropy

\[ \sigma_1 = \sigma_3 \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_w) \sin 2\beta} \]

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Recall: Yield surface

Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia
Shear enhanced compaction

Porosity loss in sandstone

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Cam-Clay model

\[ M^2 p^2 - M^2 p_0 p + q^2 = 0 \]

with

\[ p = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \]

\[ q^2 = \frac{1}{2} \left( (S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_1 - S_3)^2 \right) \]

\[ M = \frac{q}{p} \]
Cam-Clay model
Sandia geomodel (Kayenta)