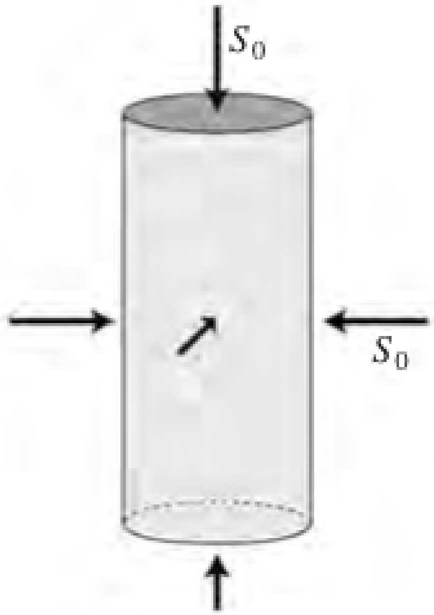
Rock failure

Types of tests on rocks

Hydrostatic compression



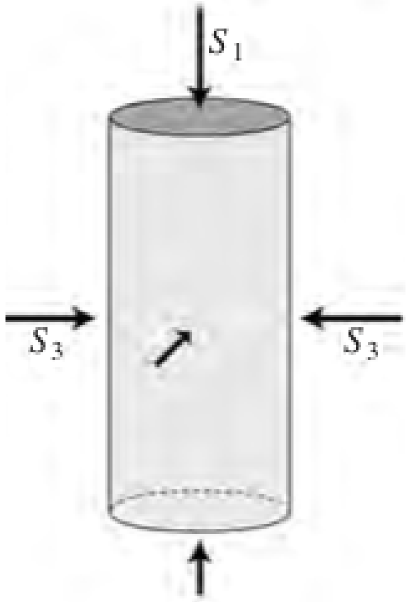
$$S_0 = S_1 = S_2 = S_3$$

Uniaxial compression

$$S_0 \neq 0 \quad S_2 = S_3 = 0$$

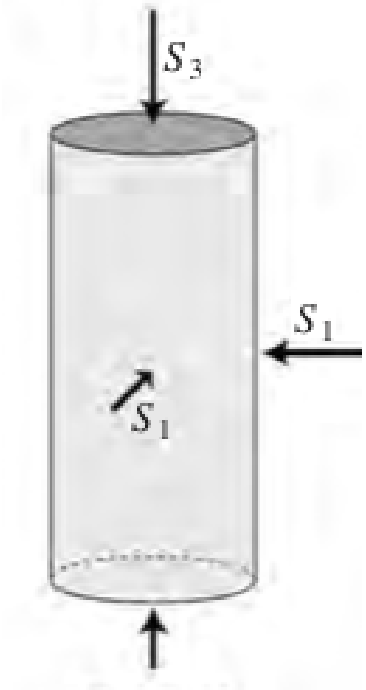
 Zoback, *Reservoir Geomechanics* (Fig. 4.1, pp. 86)

Triaxial compression



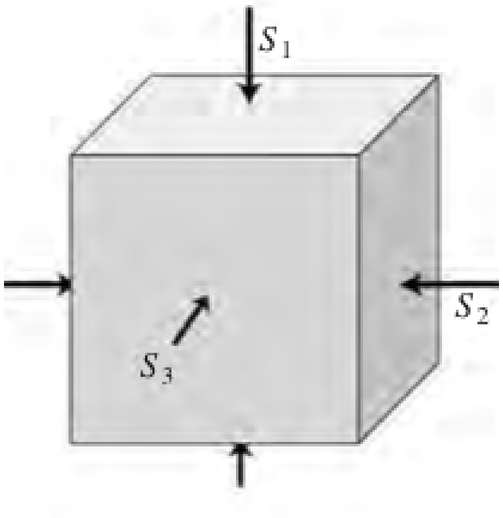
$$S_1 > S_2 = S_3$$

Triaxial extension



$$S_1 = S_2 > S_3$$

True triaxial

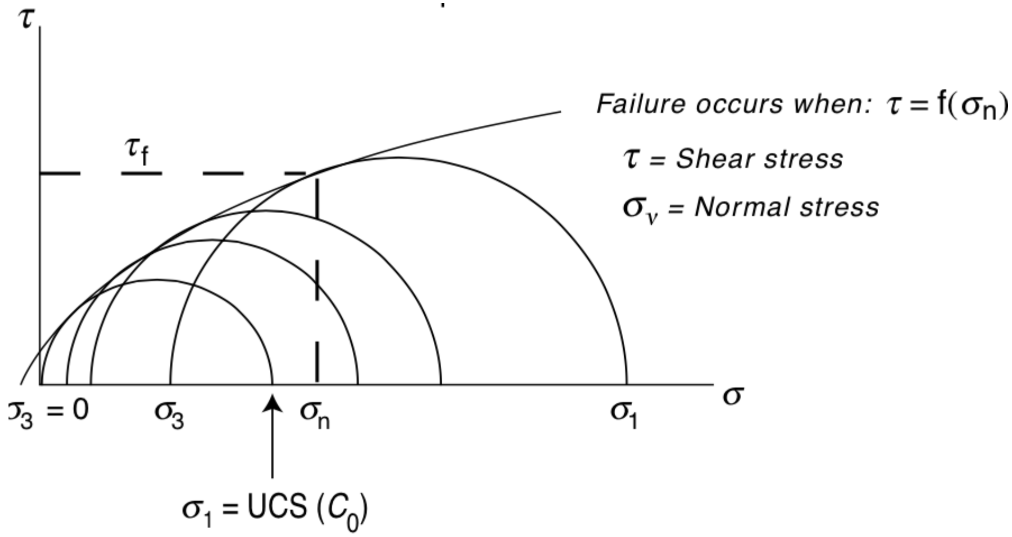


$$S_1 \neq S_2 \neq S_3$$

Mohr's circles

$$\tau_f = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(2\beta)$$
$$\sigma_n = \frac{1}{2}(\sigma + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\beta)$$

Mohr Envelope



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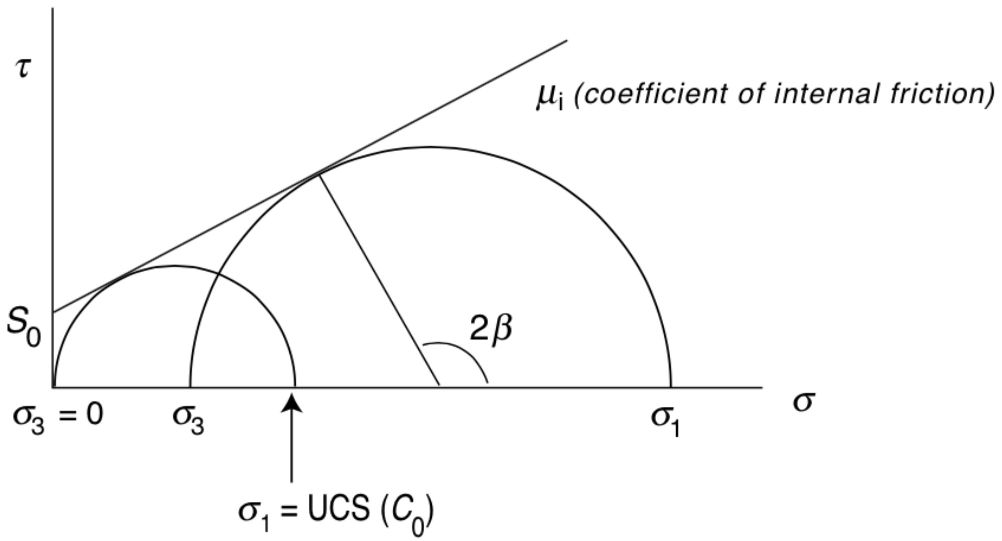
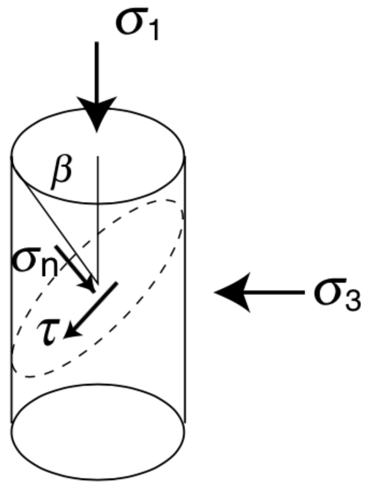
Linearized Mohr Envelope





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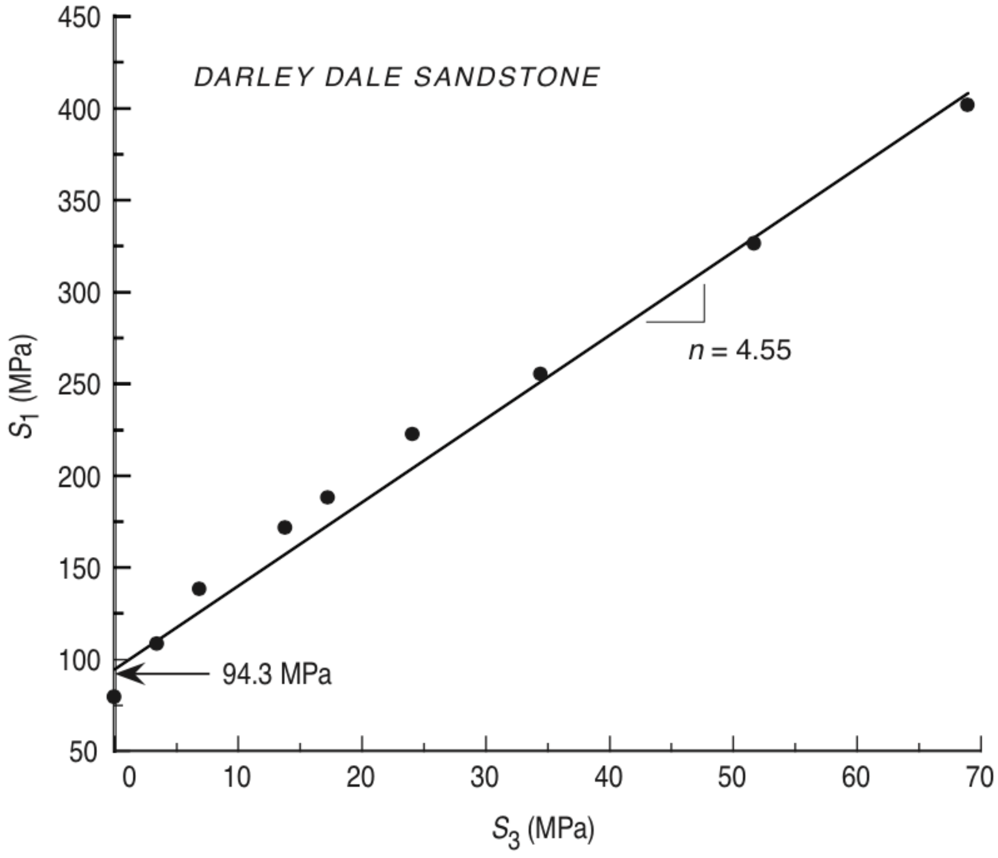
Mohr-Coulomb failure

$$\tau = S_0 + \sigma_n \mu_i$$

$$C_0 = 2S_0 \left(\sqrt{\mu_i^2 + 1} + \mu_i \right)$$

Triaxial tests on sandstone



$$\mu_i = \frac{n-1}{2\sqrt{n}}$$

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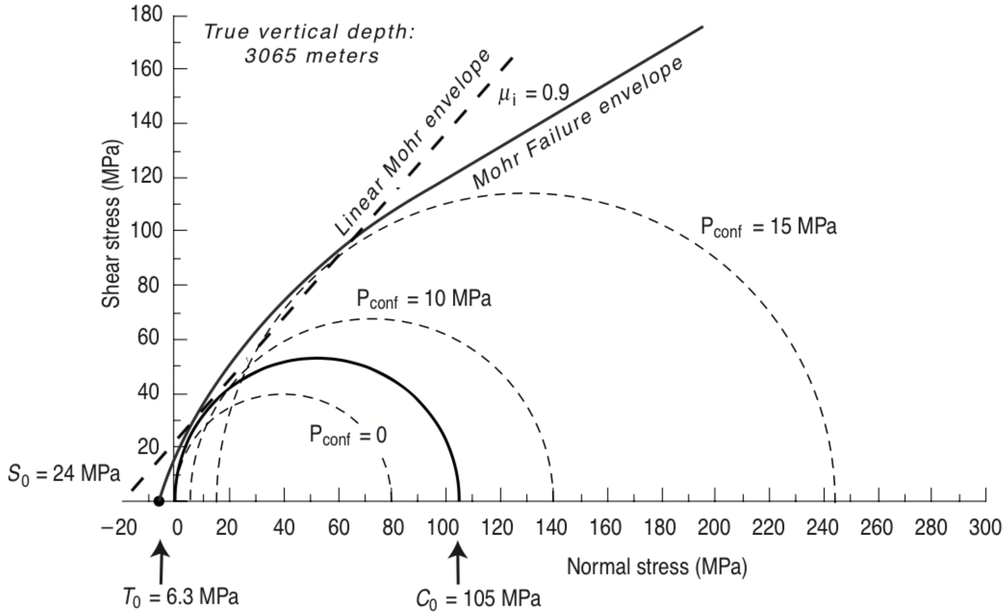
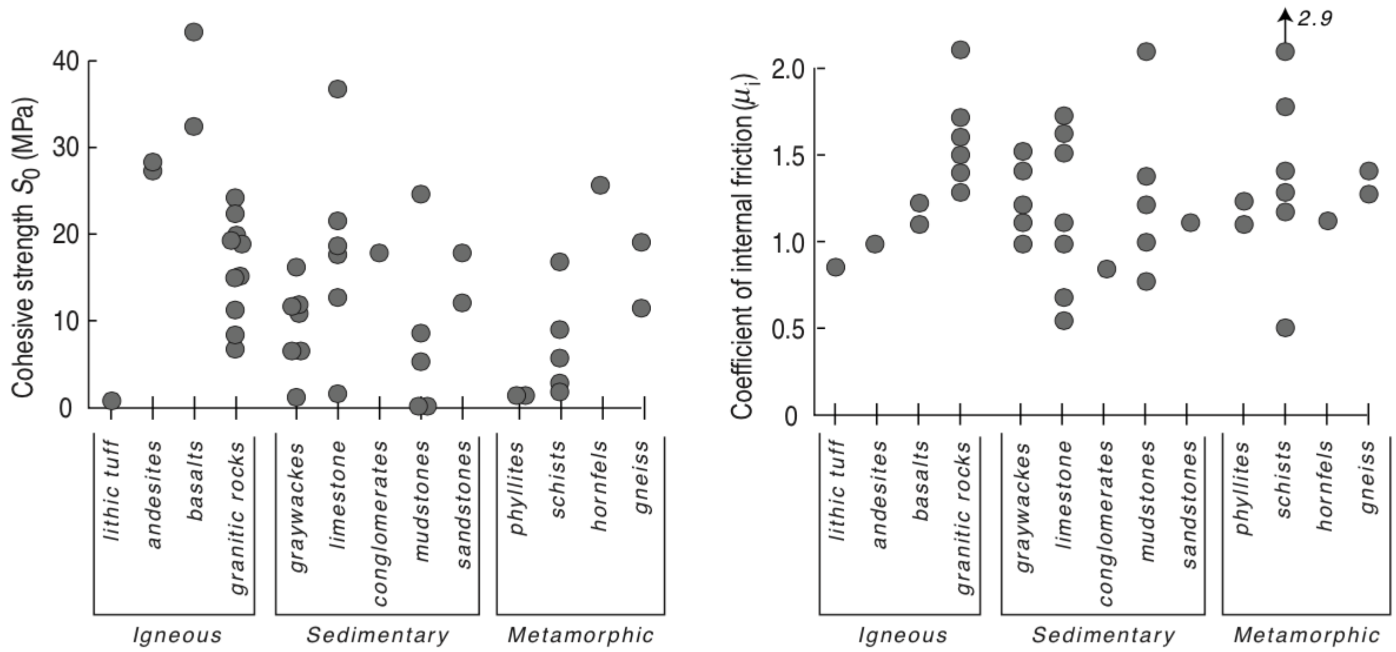
Mohr Envelope for Sandstone



 Zoback, *Reservoir Geomechanics* (Fig. 4.3a, pp. 88)

Cohesion and internal friction data



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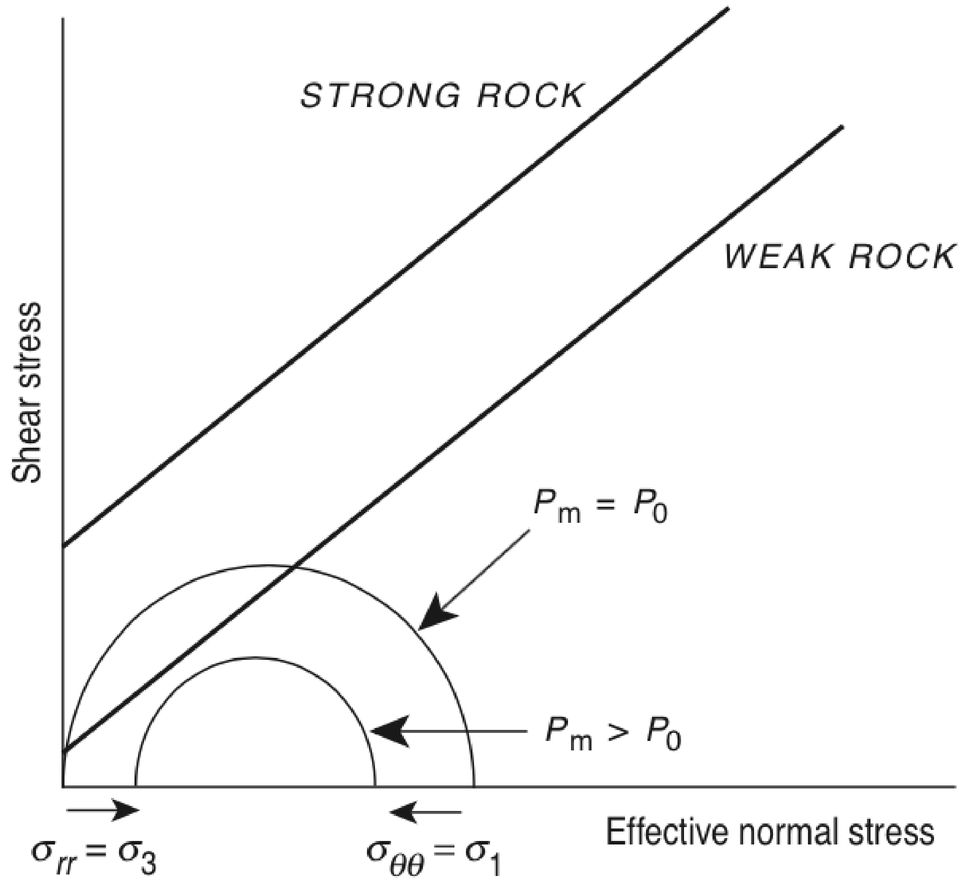
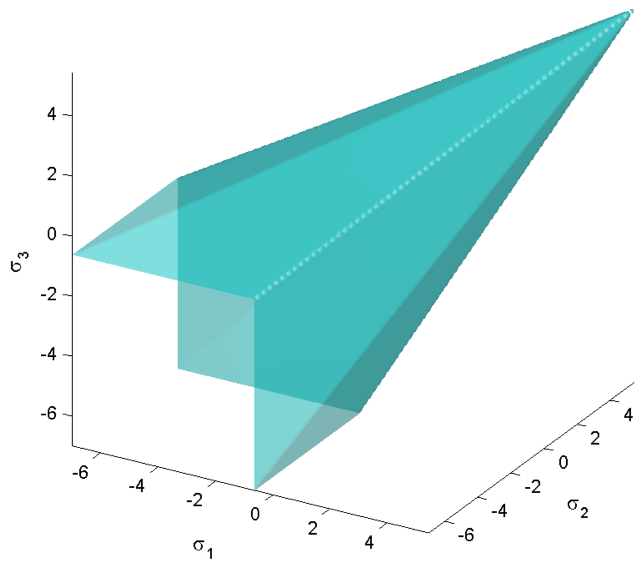
Cohesion and internal friction data



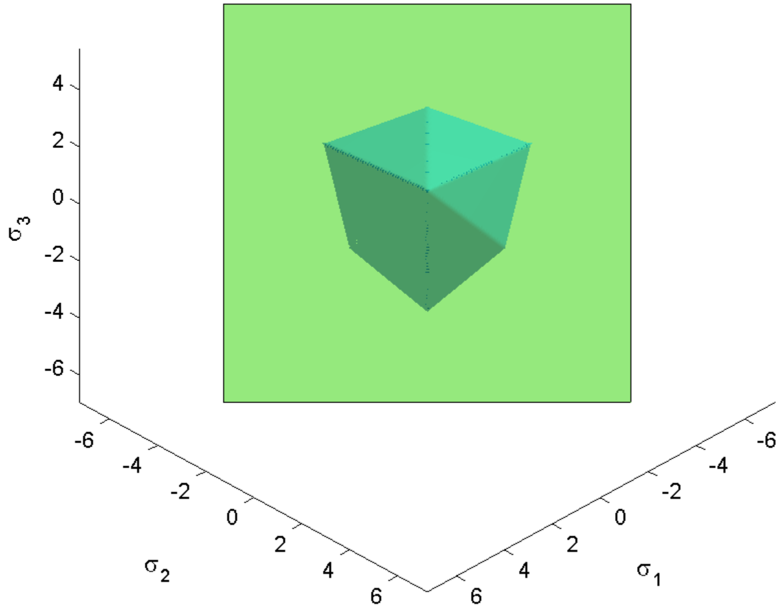
 Zoback, *Reservoir Geomechanics* (Fig. 4.5, pp. 92)

Yield surface



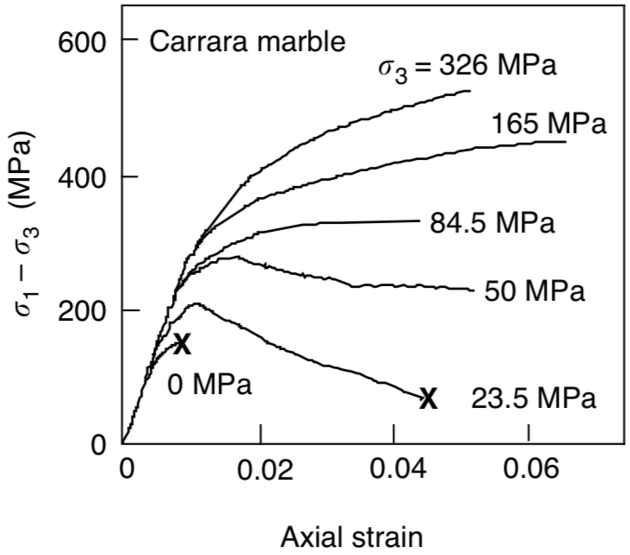
Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia

π-plane



Mohr Coulomb Yield Surface 3Db. Licensed under CC BY-SA 3.0 via Wikipedia

Pressure dependence



© Blackwell Publishing Jaeger, et al., *Fundamentals of Rock Mechanics* (Fig. 4.5, pp. 86)

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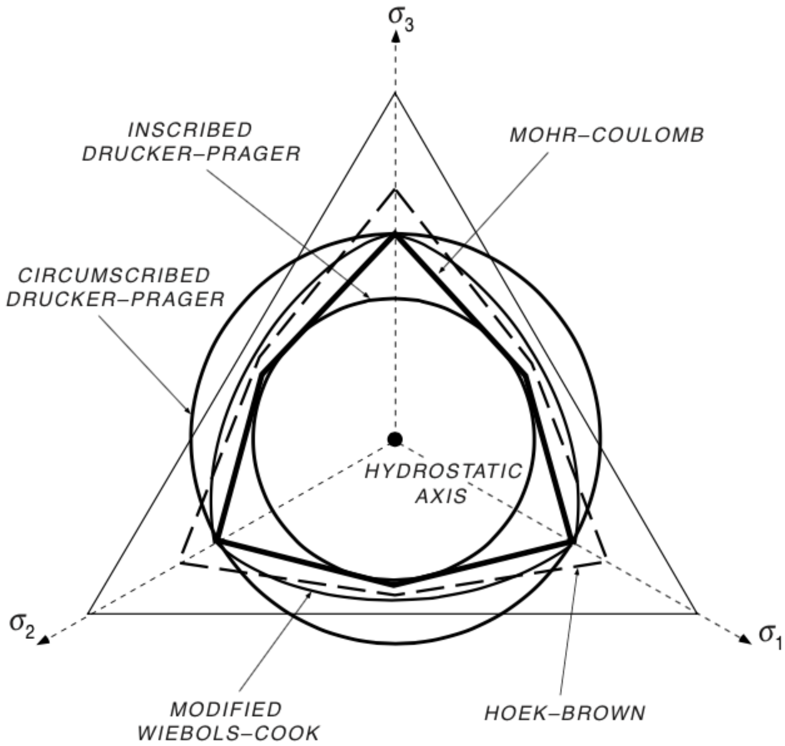
Other failure criteria



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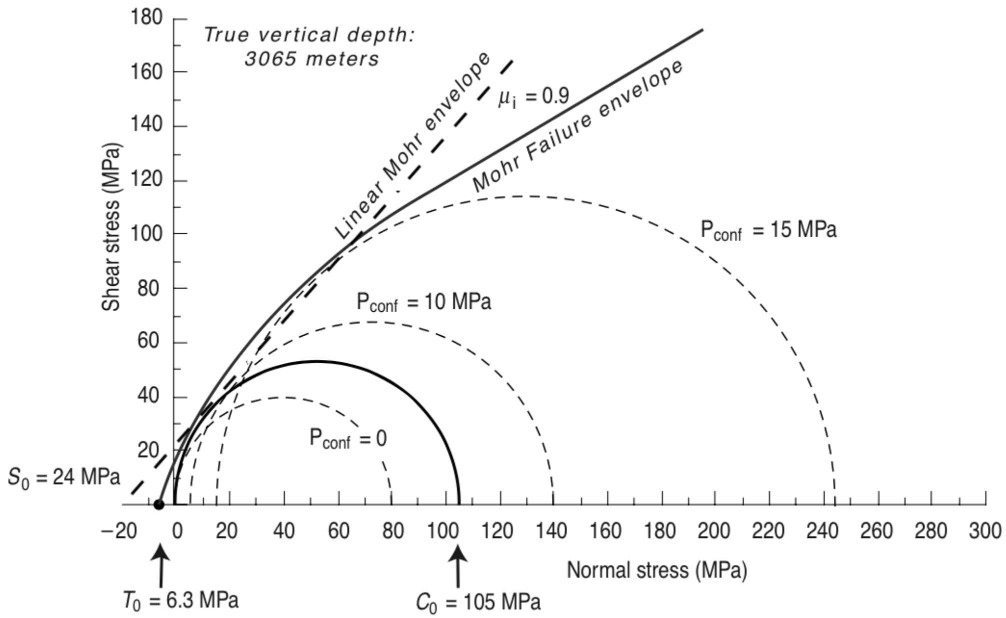
Recall: Mohr Envelope for Sandstone



 Zoback, *Reservoir Geomechanics* (Fig. 4.3a, pp. 88)

Hoek-Brown criterion (parabolic fitting)

$$\sigma_1 = \sigma_3 + C_0\sqrt{m\frac{\sigma_3}{C_0} + s}$$

m and s are fitting parameters that depend on rock properties and the degress of fracturing.

Typical values

Typical Range of m	**Types of rocks**
$5 < m < 8$	carbonate rocks (dolomite, limestone, marble)
$4 < m < 10$	lithified argillaceous rocks (sandstones, quartizite)
$15 < m < 24$	arenaceous rocks (andesite, dolerite, diabase, rhyolite)
$22 < m < 33$	course-grained polymінerallic gineous and metamorphic (amphibolite, gabbro, gneiss, norite, quartz-diorite)

Intact Rocks -- $s \to 1$

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Completely Granualated -- $s \to 0$

Lade Criterion

$$\left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{p_a}\right)^{m'} = \eta_1$$

with

$I_1 = S_{ii} = S_1 + S_2 + S_3$ (first invariant of $\mathbf{S}$)

$I_3 = \det(\mathbf{S}) = S_1 S_2 S_3$ (third invariant of $\mathbf{S}$)

p_a is atmospheric pressure, m' and n_1 are material constants

Modified Lade Criterion (dependece on σ_2)

$$\left(\frac{(I_1')^3}{I_3'}\right) = 27 + \eta$$

with

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S)$$

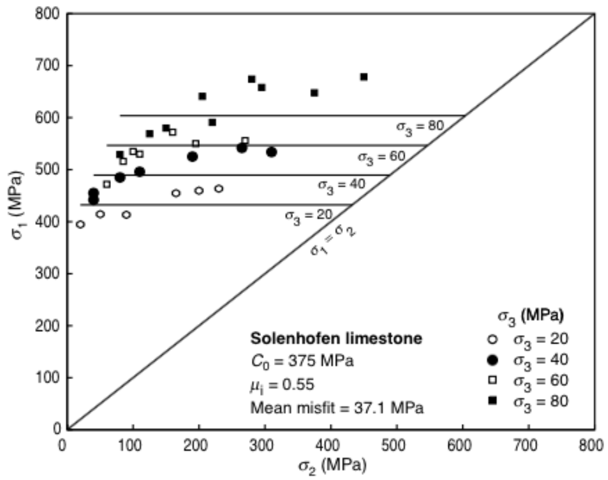
$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S)$$

$$S = \frac{S_0}{\tan\phi}$$

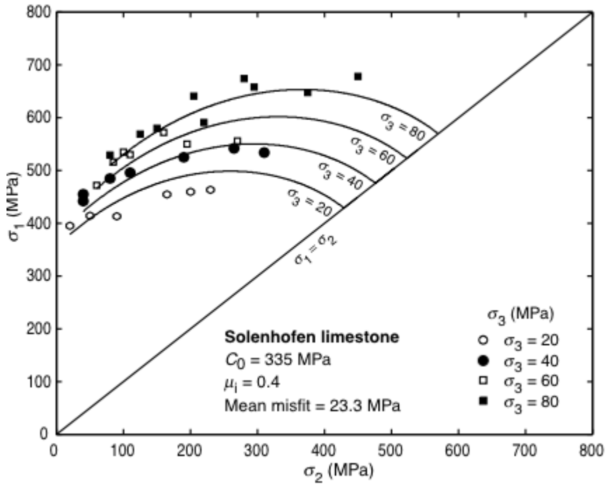
$$\eta = \frac{4(\tan\phi)^2(9 - 7\sin\phi)}{1 - \sin\phi)}$$

$\tan\phi = \mu_i$ and S_0 from Mohr-Coulomb criterion

Comparison



Mohr-Coulomb

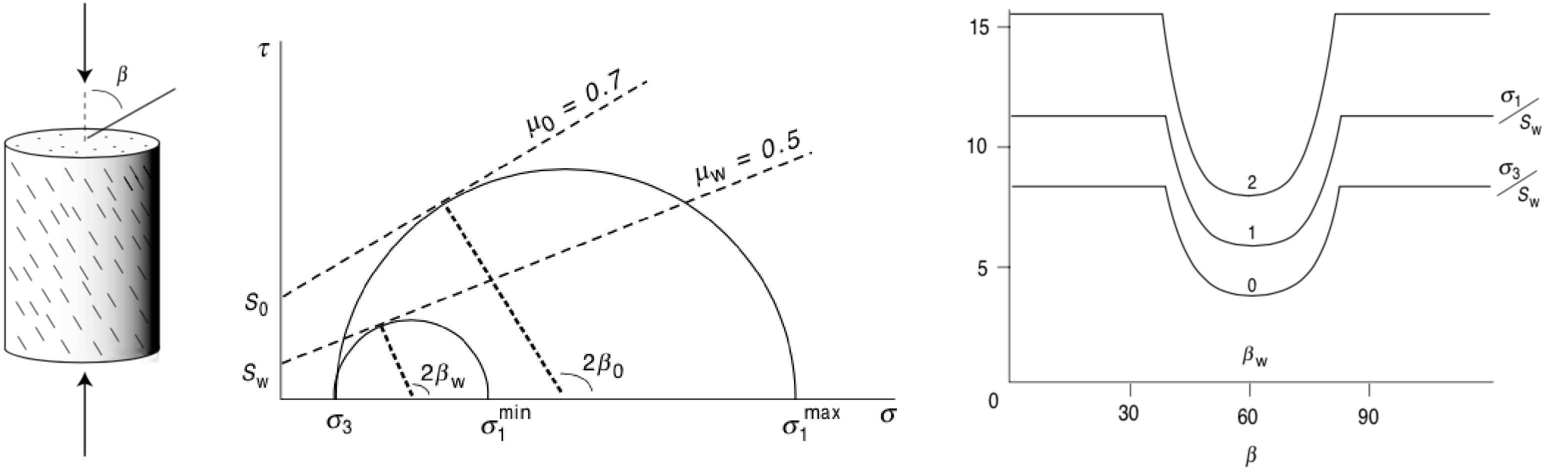


modified Lade

Others

- modified Wiebols-Cook
- Druker-Prager
- many more!

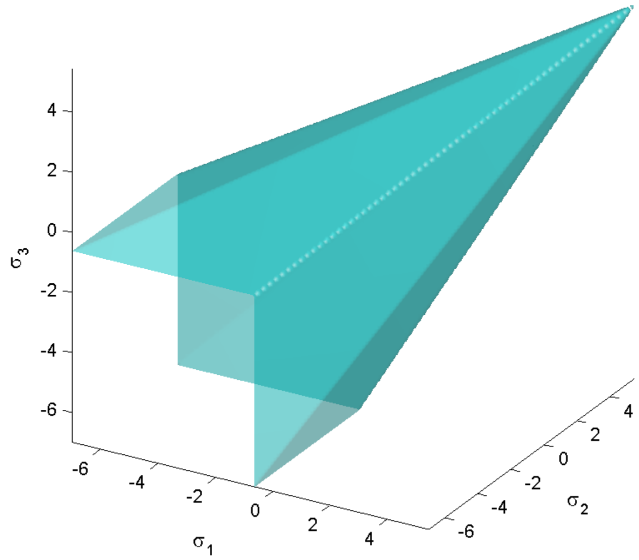
Strength anisotropy



$$\sigma_1 = \sigma_3 \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cot \beta_w) \sin 2\beta}$$

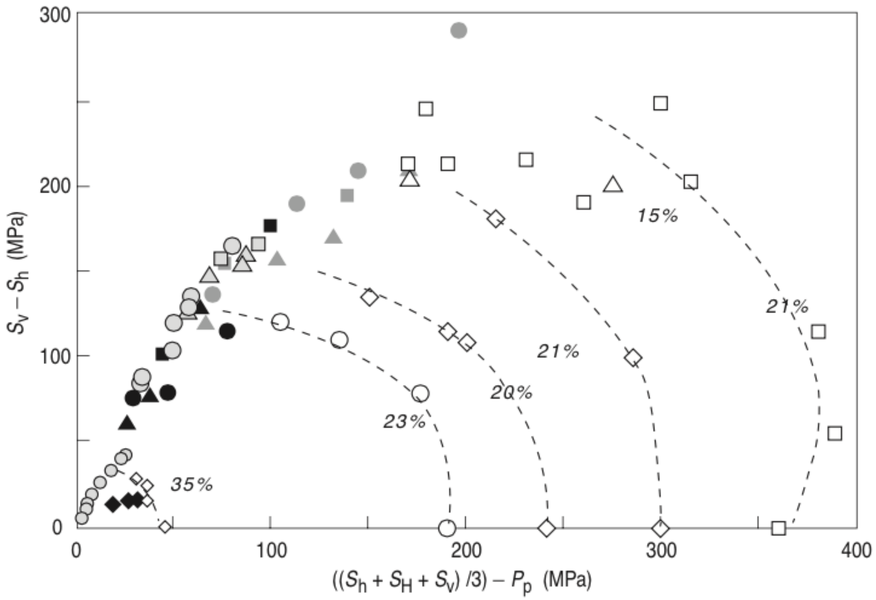
 Zoback, *Reservoir Geomechanics* (Fig. 4.12, pp. 106)

Recall: Yield surface



Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia

Shear enhanced compaction



Porosity loss in sandstone

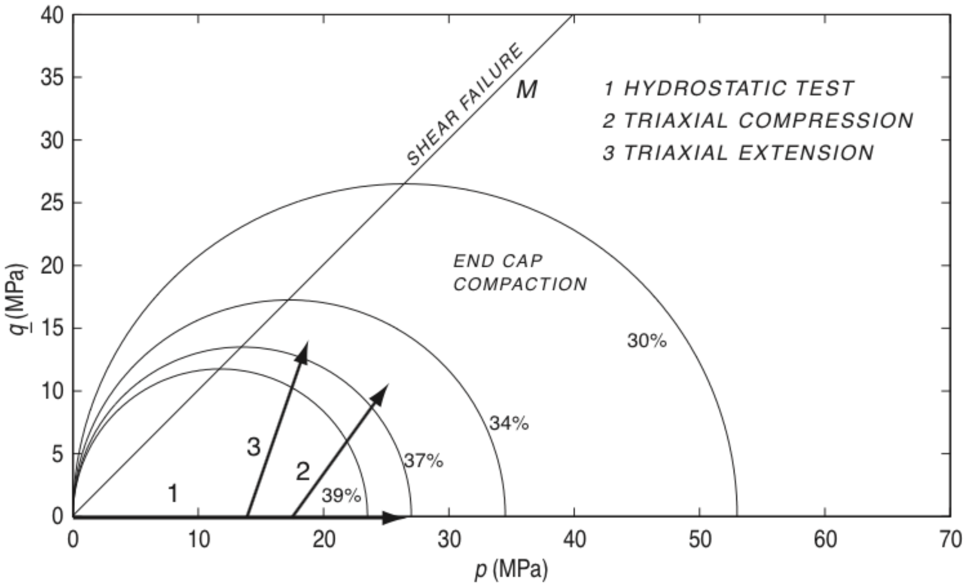
Cam-Clay model

$$M^2 p^2 - M^2 p_0 p + q^2 = 0$$

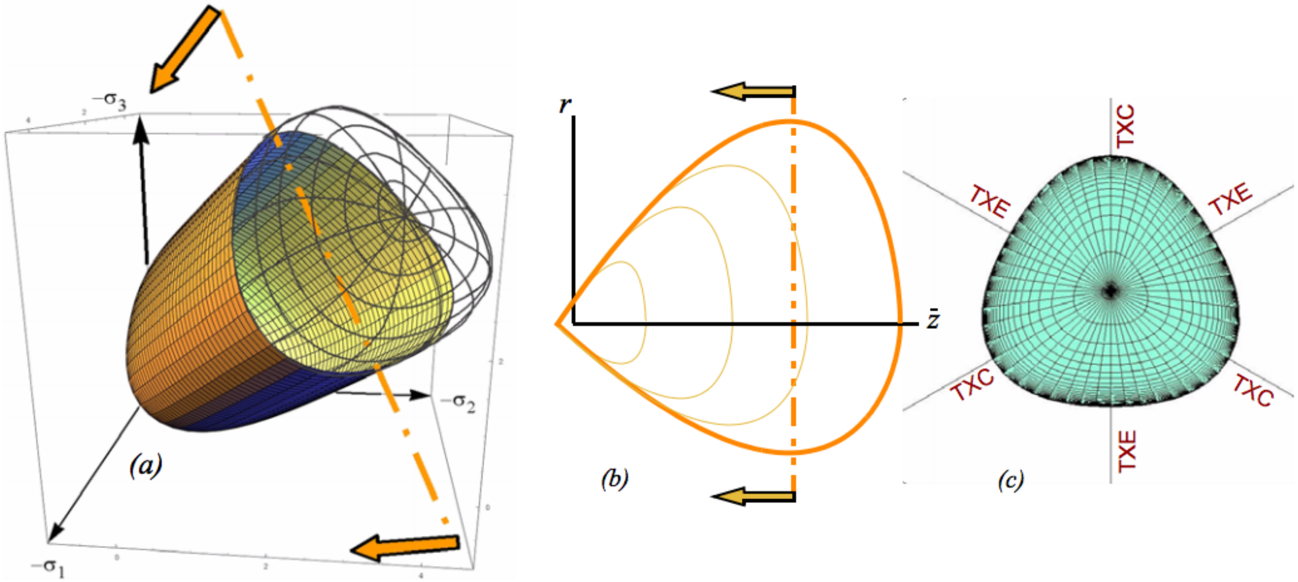
with

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$
$$q^2 = \frac{1}{2}\left((S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_1 - S_3)^2\right)$$
$$M = \frac{q}{p}$$

Cam-Clay model



Sandia geomodel (Kayenta)



R.M. Brannon, A.F. Fossum, and O.E. Strack: Kayenta: Theory and User's Guide. Tech. rep. Sandia National Laboratories, 2009.