

Homework Assignment 1

1. The motion of a certain continuous medium is defined by the equations

$$\begin{aligned}x_1 &= \frac{1}{2}(X_1 + X_2)e^t + \frac{1}{2}(X_1 - X_2)e^{-t}, \\x_2 &= \frac{1}{2}(X_1 + X_2)e^t - \frac{1}{2}(X_1 - X_2)e^{-t}, \\x_3 &= X_3\end{aligned}$$

- a. Compute the following
- The Green-Lagrange strain tensor E
 - The linear (small) strain tensor ε

Plot the 11, 22, and 12 components of E and ε on the same figure from time $t = 0$ to $t = 0.05$.

- b. Compute the following
- The rate-of-deformation tensor D
 - The rate-of-change of the small strain tensor $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$

Plot the 11, 22, and 12 components of D and $\dot{\varepsilon}$ on the same figure from time $t = 0$ to $t = 0.05$.

2. Given the following stress tensor

$$\sigma = \begin{bmatrix} 36 & 27 & 0 \\ 27 & -36 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

Find:

- the components of the traction vector acting on a plane with unit normal vector $\hat{n}^T = [2/3, -2/3, 1/3]$
- the magnitude of the traction vector found in (a)
- its component in the direction of the normal
- the angle between the traction vector and the normal

3. Given the following stress tensor

$$\sigma = \begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix}$$

Find:

- the principle stresses $\sigma_I, \sigma_{II}, \sigma_{III}$

- b. the three invariants I_1, I_2, I_3
- c. the deviatoric stress
- d. the two nonzero invariants of the deviatoric stress, i.e. J_2, J_3

4. Show that

$$\frac{\partial J_2}{\partial \sigma_{ij}} = S_{ij}$$

where J_2 is the second invariant of the deviatoric stress tensor, S_{ij} .

5. For each of the following stress states (values not given are zero), plot the three Mohr's circles and determine the maximum shear stress.
- a. Uniaxial tension $\sigma_{11} = 40$
 - b. Biaxial stress $\sigma_{11} = -10, \sigma_{22} = 30$
 - c. Hydrostatic tension of magnitude 100 psi
 - d. $\sigma_{11} = -60, \sigma_{22} = 100, \sigma_{33} = 40$
 - e. $\sigma_{11} = 10, \sigma_{22} = 40, \sigma_{21} = \sigma_{12} = 20$