assignment3

April 10, 2025

Problem 1

Consider the following equation which describes the transverse deflection associated with a simply supported beam subject to a uniform transverse load $q(x) = q_0$,

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[EI\frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2}\right] = q_0 \quad \text{for} \quad 0 < x < L,$$

subject to boundary conditions,

$$w(0) = EI\frac{\mathrm{d}^2 w(0)}{\mathrm{d}x^2} = 0, \quad \text{and} \quad w(L) = EI\frac{\mathrm{d}^2 w(L)}{\mathrm{d}x^2} = 0.$$

(a) 10 points

Develop and clearly indicate the weak form of this differential equation.

(b) 5 points

Using the following approximation for u,

$$u \approx u_h = c_1 x\left(x - L\right),$$

use the Ritz method to determine the coefficient c_1.

(c) 5 points

Using the following approximation for u,

$$u \approx u_h = c_1 \sin\left(\frac{\pi x}{L}\right)$$

use the Ritz method to determine the coefficient c_1.

(d) 5 points

Compare your answers from (b) and (c) with the analytic solution at $x = L/2$. Which is more accurate? Why? (Hint: It has something to do with the boundary conditions.)

Problem 2

We have seen that the following one-dimensional boundary value problem describes the physics of many interesting problems in engineering

$$-\frac{d}{dx}\left[a\frac{du}{dx}\right] + cu - f = 0 \quad \text{for} \quad 0 < x < L,$$

subject to the boundary conditions

$$u(0) = u_0 \quad \text{and} \quad \left[a\frac{du}{dx}\right]_{x=L} = Q_0,$$

where the x is the independent variable, $u = u(x)$ is the dependent variable, and $a = a(x), c = c(x), f = f(x), u_0$, and Q_0 are the *data* of the problem.

50 points

Write a general one-dimensional finite element code to solve this problem. General means that the user will input the *node* locations and specify the functions $a(x)$, $c(x)$, and $f(x)$ as well as the boundary conditions u_0 and Q_0. The output of the code should be the value of u at the user specified nodes. You can restrict the code to using only linear interpolated elements.

You can verify your code with following data and results:

1. For $a(x) = 1 - x/2$, $c(x) = 0$, $f(x) = 0$, $u_0 = 0$, and $Q_0 = 1$,

x	$u(x)$
0.00	0.00000
0.25	0.26666
0.50	0.57435
0.75	0.93799
1.00	1.38244

2. For $a(x) = 1 - x/2$, $c(x) = x$, $f(x) = x$, $u_0 = 0$, and $Q_0 = 1$,

x	$u(x)$
0.00	0.00000
0.40	0.46695
0.60	0.73242
0.65	0.80319
0.70	0.87615
1.00	1.38355

3. For $a(x) = 1$, $c(x) = 0$, $f(x) = x^2$, $u_0 = 0$, and $Q_0 = 2$,

x	$u(x)$
1	0.000
2	22.08
3	40.00
4	48.75