

$$\epsilon_{ENG} = \frac{\Delta L}{L_0} = \frac{L_f - L_0}{L_0} = \lambda - 1 \Rightarrow \lambda = \epsilon_{ENG} + 1$$

Engineering strain \rightarrow Lagrangian

$$\epsilon_{LOG} = \int_{L_0}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_0}\right) = \ln(\lambda) = \ln(\epsilon_{ENG} + 1)$$

Logarithmic strain, natural strain, "~~True strain~~"

$$\epsilon_{TR} = \frac{L_f - L_0}{L_f} = 1 - \frac{1}{\lambda} \rightarrow \text{Eulerian strain, "~~True strain~~"}$$

Seth-Hill

$$\epsilon_{(m)} = \frac{1}{m} (\lambda^m - 1)$$

$m = 1 \rightarrow$ Eng. strain

$m = -1 \rightarrow$ Eulerian strain

$m = 0 \rightarrow$ Log. strain

Aside

Please "label your strain"

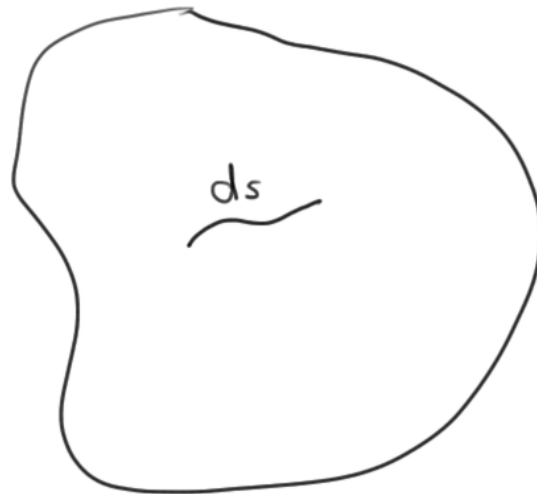
Consider $\lambda = 1.01000$

$$\epsilon_{\text{ENG}} = \lambda - 1 = 0.01000$$

$$\epsilon_{\text{LOG}} = \ln(\lambda) = 0.00990$$

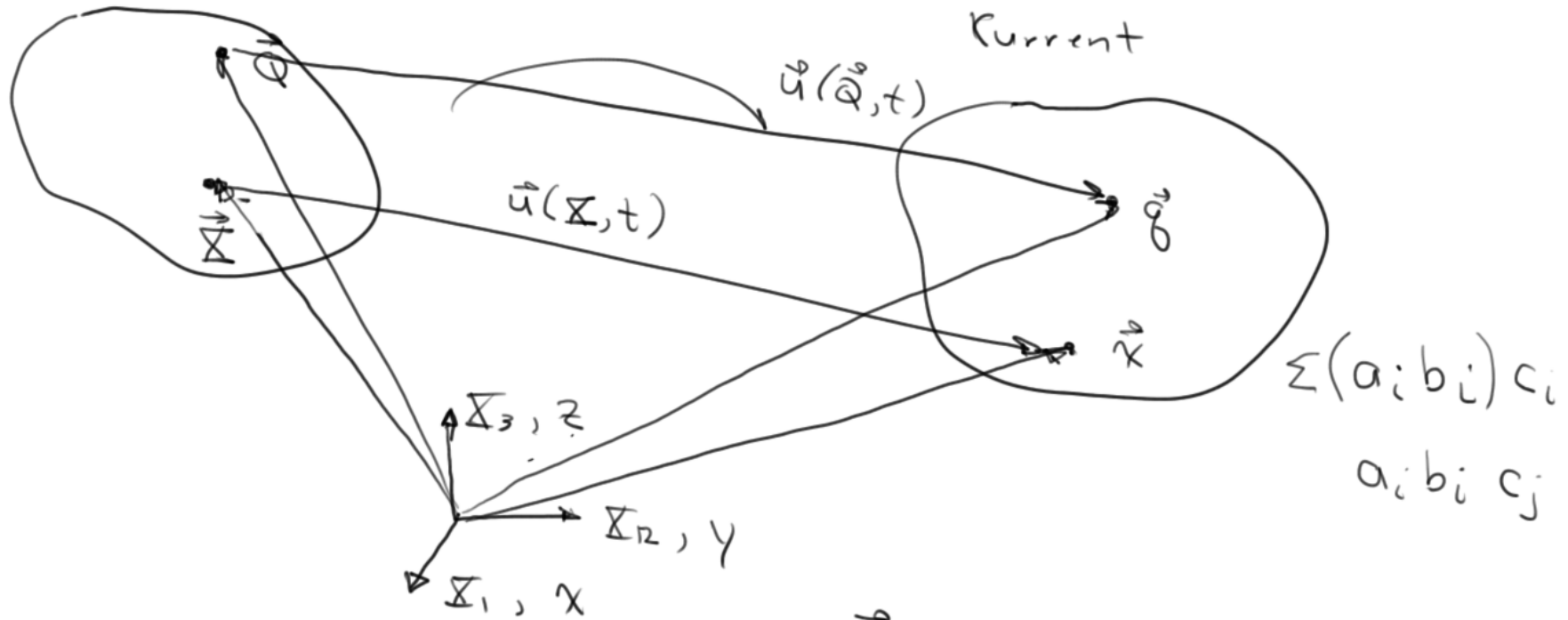


Reference
"Undeformed"
Lagrangian



Current
Deformed Eulerian

Ref.



$$\sum (a_i b_i) c_i$$

$$a_i b_i c_j$$

$$\vec{x} = \vec{X} + \vec{u}(\vec{X}, t)$$

$$\vec{q} = \vec{Q} + \vec{u}(\vec{Q}, t)$$

$$\vec{x} = \vec{x}(\vec{X}, t)$$

$$\vec{q} = \vec{q}(\vec{Q}, t)$$

$$\vec{X} = X_1 \hat{i} + X_2 \hat{j} + X_3 \hat{k}$$

$$\vec{X} = X_1 \hat{e}_1 + X_2 \hat{e}_2 + X_3 \hat{e}_3$$

$$\vec{X} = \sum_{i=1}^3 X_i \hat{e}_i$$

$$\vec{X} = X_i \hat{e}_i$$