$$E = \frac{1}{2}\left(F^T F - I\right) \qquad F = (\nabla_X u)^T + I$$

$$F^T = (\nabla_X u) + I$$

$$= \frac{1}{2}\left[\nabla_X u + (\nabla_X u)^T + (\nabla_X u)^T(\nabla_X u)\right]$$

$$= \frac{1}{2}\left[\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \cancelto{0}{\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}}\right] \Rightarrow$$

Linear strain → Cauchy strain

$$\boxed{\varepsilon = \frac{1}{2}\left[\nabla_X u + (\nabla_X u)^T\right] = \underbrace{\frac{1}{2}\left[\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right]}}$$

$$(ds)^2 - (dS)^2 = d\vec{x}^T d\vec{x} + d\vec{X}^T d\vec{X}$$

$$d\vec{x} = F\, d\vec{X} \Rightarrow d\vec{X} = F^{-1} d\vec{x}$$

$$\|\nabla_X u\| << 1$$

$$\|A\| = \sqrt{A_{ij} A_{ij}}$$

$$(ds)^2 - (dS)^2 = d\vec{x}^T d\vec{x} - (F^{-1} d\vec{x})^T (F^{-1} d\vec{x})$$

$$= d\vec{x}^T d\vec{x} - d\vec{x}^T (F^{-T}) F^{-1} d\vec{x}$$

$$= d\vec{x}^T \underbrace{\left(I - F^{-T} F^{-1}\right)}_{2e} d\vec{x}$$

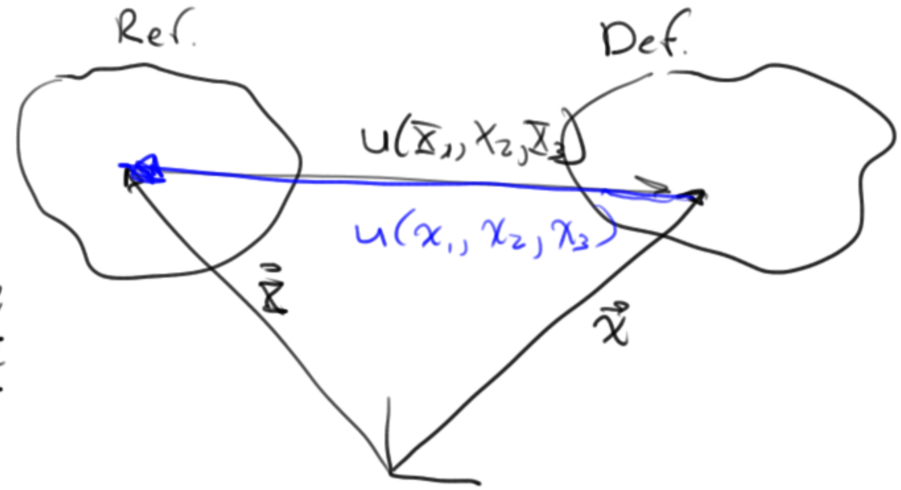
$$e = \frac{1}{2}\left[I - F^{-T}F^{-1}\right]$$ ← Eulerian strain, Almansi strain

$$e = \frac{1}{2}\left[I - F^{-T}F^{-1} \right]$$

$$= \frac{1}{2}\left[\nabla_{\vec{x}} u + (\nabla_{\vec{x}} u)^T + (\nabla_{x} u)^T (\nabla_{x} u) \right]$$

$$= \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \cancelto{0}{\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}} \right]$$

$$F = \frac{\partial \vec{x}}{\partial \vec{X}}$$



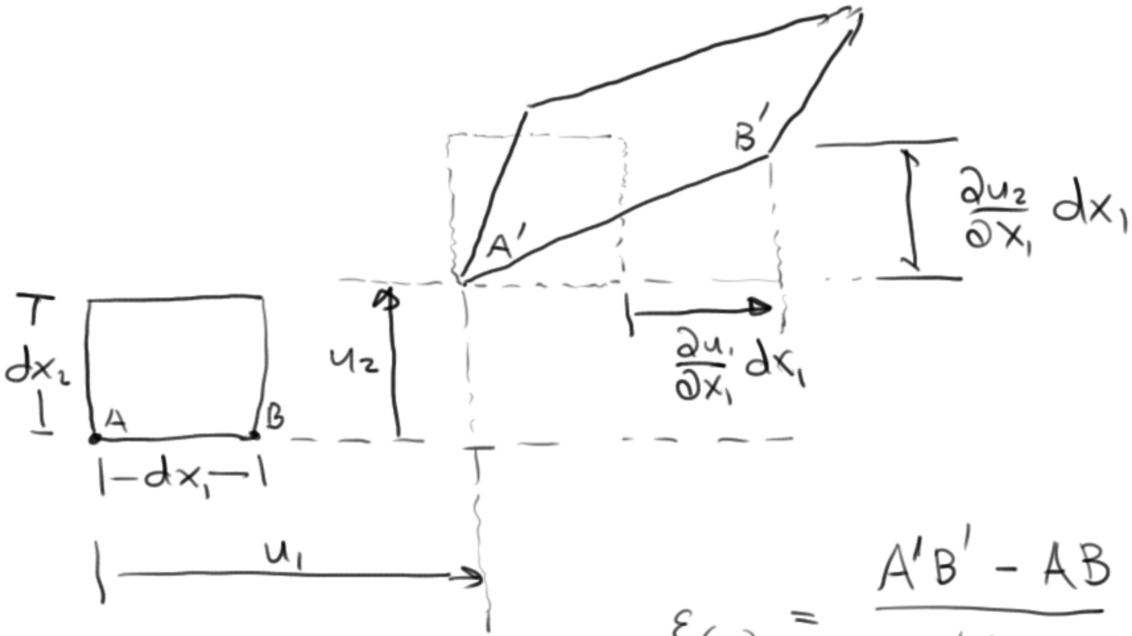
$$\left[\frac{\partial \vec{X}}{\partial \vec{x}}\right] = \frac{\partial \vec{x}}{\partial \vec{x}} + \frac{\partial \vec{u}(x_1, x_2, x_3)}{\partial \vec{x}}$$

$$\boxed{F^{-1} = I + (\nabla_{\vec{x}} \vec{u})^T}$$

$$\varepsilon = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]$$

$$\vec{x} = \vec{X} + \vec{u}(\cancelto{\approx 0}{\vec{X}})$$

$$\vec{x} \approx \vec{X}$$



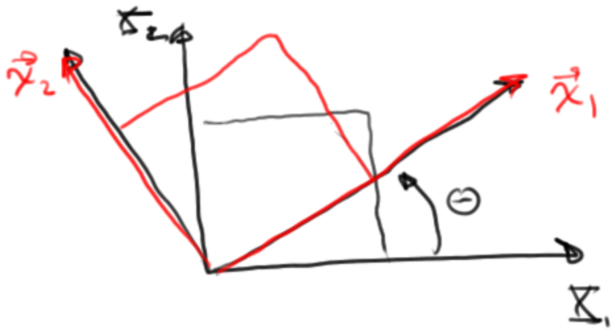
$$\varepsilon_{(x_1)} = \frac{A'B' - AB}{AB} = \frac{A'B' - dx_1}{dx_1} = \frac{A'B'}{dx_1} - 1$$

$$(A'B')^2 = \left[\left(dx + \frac{\partial u_1}{\partial x_1} dx_1 \right)^2 + \left(\frac{\partial u_2}{\partial x_1} dx_1 \right)^2 \right]$$

$$\varepsilon_{(x_1)}^2 = \left(\frac{A'B'}{dx_1} + 1 \right)^2$$

$$\cancel{\varepsilon_{(x_1)}^2}^{\,0} = \underbrace{\cancel{2}\varepsilon_{(x_1)} + \cancel{1} = \cancel{1} + \cancel{2}\frac{\partial u_1}{\partial x_1}} + \left(\cancel{\frac{\partial u_1}{\partial x_1}}^{\,0} \right)^2 + \left(\cancel{\frac{\partial u_2}{\partial x_1}}^{\,0} \right)^2$$

$$\varepsilon_{(x_1)} = \frac{\partial u_1}{\partial x_1} \longleftarrow \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = \cancel{\frac{1}{2}} \left(\cancel{2} \frac{\partial u_1}{\partial x_1} \right)$$



$$\begin{Bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

$$d\vec{x} = F \, d\vec{X}$$

$$F = R$$

$$E = \frac{1}{2}\left[F^T F - I\right] = \frac{1}{2}\left[R^T R - I\right]$$

$$= \frac{1}{2}\left[R^{-1} R - I\right]$$

$$= \frac{1}{2}\left[I - I\right] = \underline{\underline{0}}$$

$$F = (\nabla u^T) + I \Rightarrow \nabla u^T = F - I$$
$$\nabla u = F^T - I$$

$$\varepsilon = \frac{1}{2}(\nabla u + \nabla u^T) = \frac{1}{2}\left[F - I + (F - I)^T\right] = \frac{1}{2}\left[F + F^T\right] - I$$

$$= \frac{1}{2}\left[R + R^T\right] - I$$

$$\neq \underline{0} \quad \text{for any } R \neq I$$