Strain-rate

$$\frac{d}{dt}(d\vec{x}) = d\left(\frac{d\vec{x}}{dt}\right) = d\vec{v}$$

$\vec{v}$

$$F = I + (\nabla u)^T$$

$$\frac{\partial x}{\partial X} = \frac{\partial u}{\partial X} + \frac{\partial X}{\partial X}$$

$$(\nabla u)^T + I$$

$$\vec{v} = \vec{v}(\vec{x}_1, \vec{x}_2, \vec{x}_3, t)$$

$$dv_i = \underbrace{\frac{\partial v_i}{\partial x_j}}\, dx_j$$

$L \rightarrow$ velocity gradient

Malvern

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$$\frac{d}{dt} F_{ij} = \frac{d}{dt} \frac{\partial x_i}{\partial X_j}$$

$$\dot{F} F^{-1} = \frac{\partial v_i}{\cancel{\partial X_j}} \cdot \underbrace{\frac{\cancel{\partial X_j}}{\partial x_k}}_{F^{-1}} = \frac{\partial v_i}{\partial x_k} = \boxed{L = \dot{F} F^{-1}}$$

$$\frac{d}{dt}\left[(ds)^2 - \cancel{(dS)^2} \right] = \underbrace{d\vec{X}^T (2E)\, d\vec{X}}\quad \frac{d}{dt}\left[\underbrace{d\vec{x}^T d\vec{x} - d\vec{X}^T \cancel{d\vec{X}}} \right] = d\vec{X}^T (\underbrace{F^T F - I}_{2E})\, d\vec{X}$$

$$\frac{d}{dt}(ds)^2 = \frac{d}{dt}\left(d\vec{x}^T d\vec{x} \right)$$

$$= d\left(\cancel{\frac{d\vec{x}^T}{dt}}^{\,v^T} \right) d\vec{x} + d\vec{x}^T d\left(\cancel{\frac{d\vec{x}}{dt}}^{\,v} \right)$$

$$= d\vec{v}^T d\vec{x} + d\vec{x}^T d\vec{v}$$

$$= (L\, d\vec{x})^T d\vec{x} + d\vec{x}^T (L\, d\vec{x})$$

$$= d\vec{x}^T L^T d\vec{x} + d\vec{x}^T L\, d\vec{x}$$

$$= d\vec{x}^T (\underbrace{L^T + L}_{2D})\, d\vec{x}$$

$$D = \frac{1}{2}\left(L^T + L \right) \rightarrow \text{rate-of-deformation tensor}$$

$$= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \qquad \dot{\varepsilon} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial X_j} + \frac{\partial \dot{u}_j}{\partial X_i} \right) = \frac{1}{2}\left(\frac{\partial v_i}{\partial X_j} + \frac{\partial v_j}{\partial X_i} \right)$$

$$\vec{x} = \vec{X} + \vec{u}$$

$$\frac{d}{dt}(ds)^2 = \cancel{d\vec{X}}^T\left(\cancel{2}\,\frac{dE}{dt}\right)\cancel{d\vec{X}}$$

$$\frac{d}{dt}(ds)^2 = d\vec{x}^T(2D)\,d\vec{x}$$

$$= (F\,d\vec{X})^T(2D)(F\,d\vec{X})$$

$$= \cancel{d\vec{X}}^T F^T(\cancel{2}D)F\,\cancel{d\vec{X}}$$

$$\boxed{d\vec{x} = F\,d\vec{X}}$$

$$\boxed{\frac{dE}{dt} = F^T D F}$$ → rate-of-Green-strain

Eulerian strain-rate

$$\frac{d}{dt}(ds)^2 = \frac{d}{dt}\left(d\vec{x}^T(2e)\,d\vec{x}\right)$$

$$= 2\left[(L\,d\vec{x})^T e\,d\vec{x} + d\vec{x}\,\frac{d}{dt}e\,d\vec{x} + d\vec{x}\,e\,(L\,d\vec{x})\right]$$

$$= \cancel{2}\,d\vec{x}\,(\underbrace{L^T e + \dot{e} + eL}_{\cancel{2}D})\,d\vec{x}$$

$$\boxed{\dot{e} = D - L^T e - eL}$$ → rate-of-Eulerian strain

Stress (Heuristic argument)

$$P(1) = \frac{dW}{dt} = \int \vec{f} \cdot \frac{d\vec{x}}{dt} \left(\frac{dV}{dV}\right)$$

$\frac{d\vec{x}}{dt} \rightarrow \vec{v}$

$\vec{x} = \vec{x}(\vec{X}, t)$ "current" configuration

$$= \int \underbrace{\frac{f_i}{A_j}}_{\sigma} \underbrace{\frac{dv_i}{dx_j}}_{L} \, dV$$

$$dV = A \, d\vec{x} = A_j \, dx_j$$

$$= \int \sigma : (D + W) \, dV$$

$$L = \text{symm.}(L) + \text{antisymm}(L)$$

$$= \underbrace{\tfrac{1}{2}(L^T + L)}_{} + \underbrace{\tfrac{1}{2}(L - L^T)}_{}$$

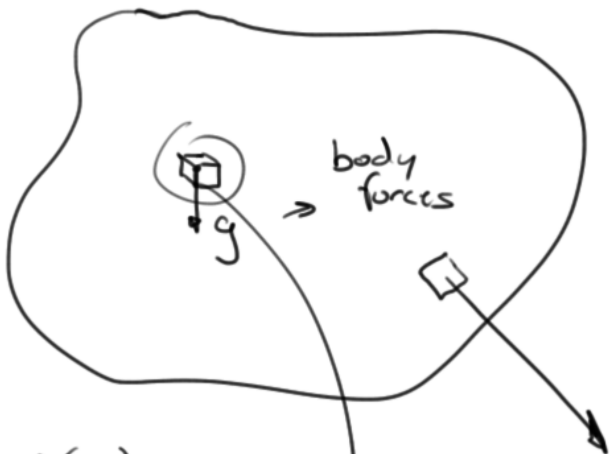
$$= \quad D \quad + \quad W$$

$$\sigma : D = \sigma_{ij} D_{ij}$$

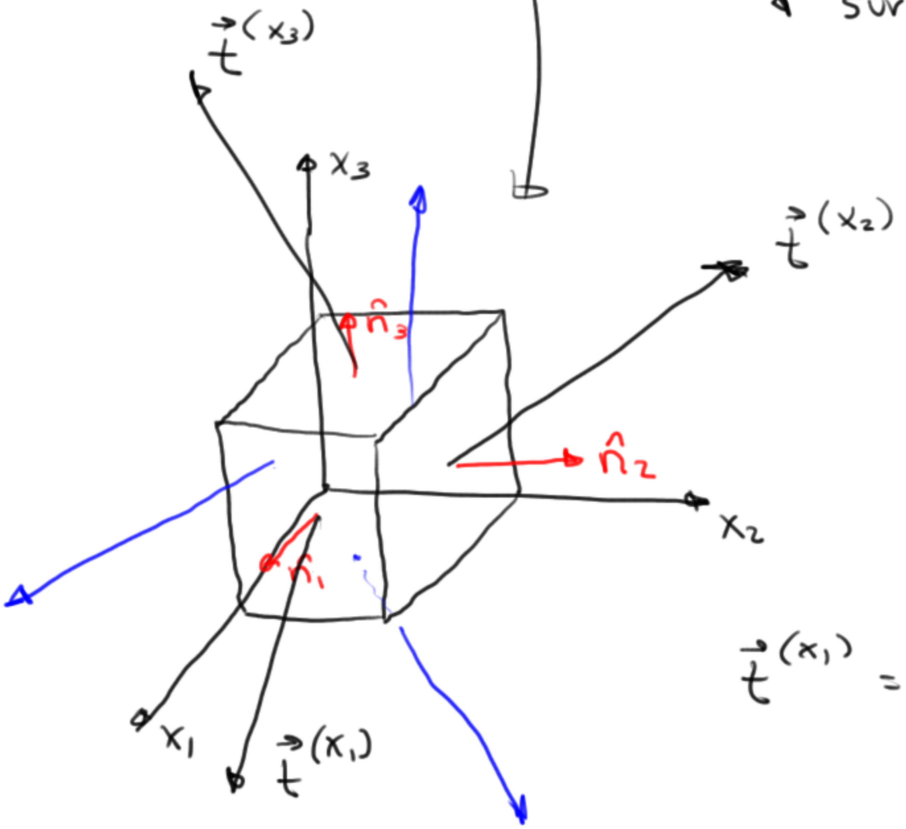
$$\sigma : W = 0$$

$$P = \int \sigma : D \, dV$$

Work (Power) - conjugate



surface forces → tractions → $\frac{\text{force}}{\text{area}}$



$$\vec{t}^{(x_1)} = \begin{Bmatrix} t_1^{(x_1)} \\ t_2^{(x_1)} \\ t_3^{(x_1)} \end{Bmatrix}$$