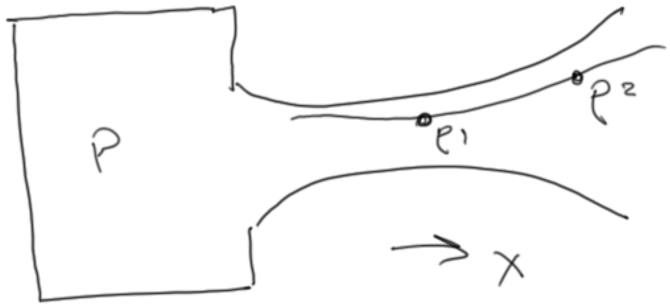


$$\frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial x} \neq 0$$

$$\vec{v} = \vec{v}\left(\vec{x}(X,t),t\right)$$

$$\frac{D}{Dt}(\vec{v}) = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x_k} v_k$$

$$= \underline{\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}}$$

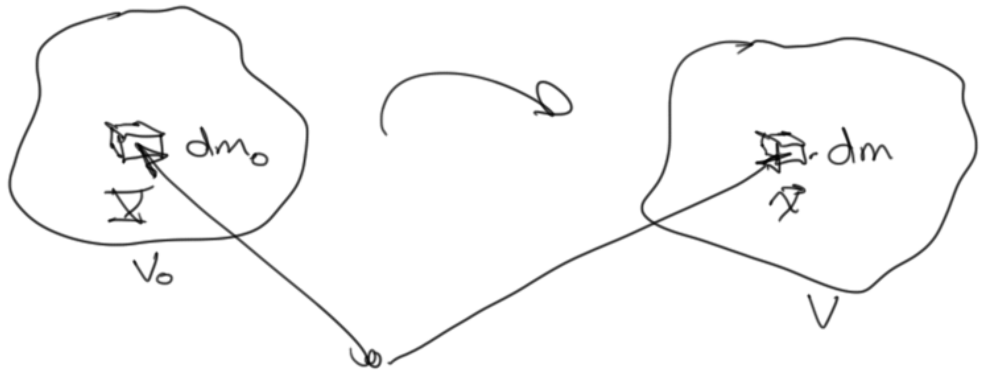
$$\frac{D}{Dt}(\vec{v})$$

$$\frac{\partial v}{\partial t}$$

Mass conservation (Material Form)

$$\frac{D}{Dt}(\cdot) = \frac{\partial}{\partial t}(\cdot) + v_i \frac{\partial}{\partial x_i}(\cdot)$$
$$= \frac{\partial}{\partial t}(\cdot) + \vec{v} \cdot \nabla(\cdot)$$

Material time derivative



$$dm_0 = \rho_0 \, dV_0$$

$$\int_{V_0} \rho_0 \, dV_0 = \int \rho \, dV$$

$$dV_0 = dX_1 \, dX_2 \, dX_3$$
$$dV = dx_1 \, dx_2 \, dx_3$$

$$dV = |J| \, dV_0 = \left| \det\left(\underbrace{\frac{\partial \vec{x}_i}{\partial \vec{X}_j}} \right) \right| dV_0$$
$$\underbrace{|\det(F)|}_{J} \, dV_0$$

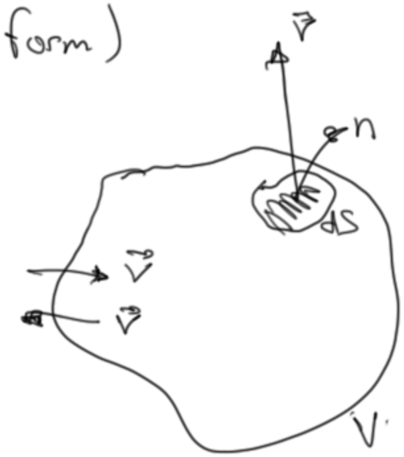
$$\det(F) = 1$$

$$\int_{V_0} \rho_0 \, dV_0 = \int \rho \, |J| \, dV_0$$

$$\rho_0 = \rho |J|$$

$$|J| = \frac{\rho_0}{\rho} \Rightarrow \boxed{J = \det(F) = \frac{\rho_0}{\rho}}$$

Mass Conservation (differential form)



Divergence Theorem

$$\boxed{\int_S \vec{v}\cdot\hat{n}\,dS = \int \nabla\cdot\vec{v}\,dV}$$

$$\text{mass} = \int \rho\,dV = \int \rho(x_i,t)\,dV$$

time rate-of-change of mass = $\underbrace{\text{mass enters} - \text{mass exits}}_{\text{mass flux}}$

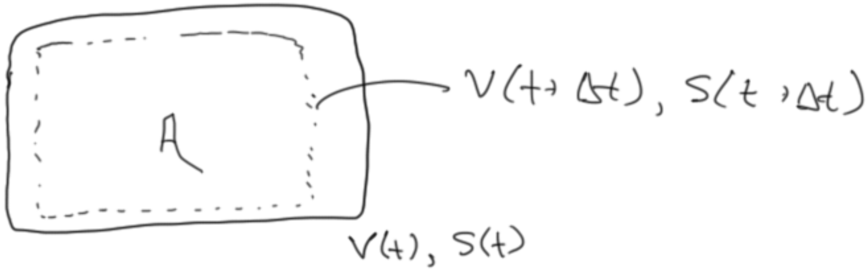
$$\frac{d}{dt}\int \rho\,dV = \underline{\int \frac{d\rho}{dt}\,dV} = -\int \rho\vec{v}\cdot\hat{n}\,dS = \underline{-\int \nabla\cdot(\rho\vec{v})\,dV}$$

$$\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0}$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho\vec{v})}{\partial x_1} + \frac{\partial(\rho\vec{v})}{\partial x_2} + \frac{\partial(\rho\vec{v})}{\partial x_3} = 0 \Rightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i)$$

$$\underbrace{\frac{\partial\rho}{\partial t} + v_i\frac{\partial}{\partial x_i}\rho} + \rho\frac{\partial\vec{v}}{\partial x_i} = 0$$

$$\frac{D\rho}{Dt} + \rho\frac{\partial\vec{v}}{\partial x_i} = 0 \Rightarrow \boxed{\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{v} = 0}$$



$$\frac{d}{dt}\int_{V(t)} \rho A \, dV$$

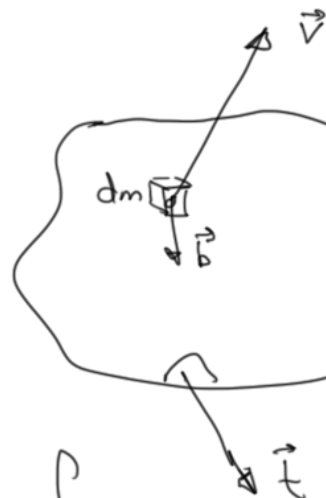
time rate-of-change of A = instantaneous change A at a constant + flux A

$$\frac{d}{dt}\int_{V(t)} \rho A \, dV = \underline{\int \frac{d}{dt}(\rho A)\, dV} + \underbrace{\int \rho A \vec{v}\cdot\vec{n}\, dS}_{\underline{\int \frac{\partial}{\partial x_i}(\rho A \vec{v})\, dV}}$$

$$= \int A \frac{d}{dt}(\rho) + \rho \frac{d}{dt}(A) + A\, \nabla\cdot(\rho\vec{v}) + \rho\vec{v}\,\nabla A \, dV$$

$$= \int A \Big(\underbrace{\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v})}_{=0}\Big) + \underbrace{\rho\frac{\partial A}{\partial t} + \rho\vec{v}\,\nabla A}_{\rho\frac{DA}{Dt}} \, dV$$

$$\boxed{\frac{d}{dt}\int_{V(t)} \rho A \, dV = \int \rho \frac{DA}{Dt} \, dV} \rightarrow \text{Reynolds Transport Theorem}$$



body forces $= \int \rho \vec{b} \, dV$

surface forces $= \int \vec{t} \, dS = \int \sigma^T \hat{n} \, dS$

$$d\vec{p} = \vec{v} \, dm = \vec{v} \rho \, dV$$

$$p = \int \rho \vec{v} \, dV$$

$$\frac{d}{dt} p = \underbrace{\frac{d}{dt}\int \rho \vec{v} \, dV}_{\text{R.T.T.}} = \int \rho \vec{b} \, dV + \underbrace{\int \sigma^T \hat{n} \, dS}_{\text{D.T.}}$$

$$\int \rho \frac{D\vec{v}}{Dt} \, dV = \int \rho \vec{b} \, dV + \int \nabla \cdot \sigma^T \, dV = \int [\rho \vec{b} + \nabla \cdot \sigma^T] \, dV$$

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho \vec{b} + \nabla \cdot \sigma^T}$$ Cauchy momentum eqn.