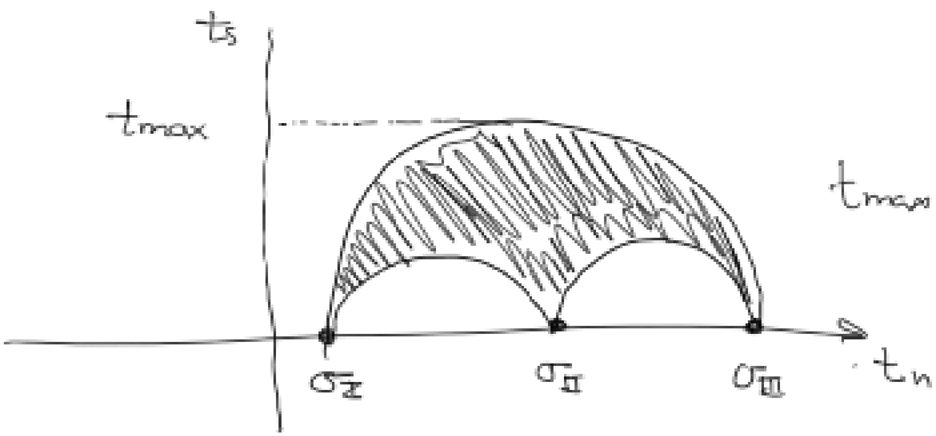
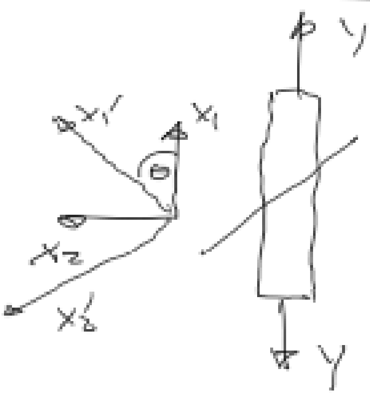
$$\left[t_n - \tfrac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\tfrac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$

$$\left[t_n - \tfrac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \geq \left(\tfrac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$



$$t_{max} = \tfrac{1}{2}(\sigma_I + \sigma_{III})$$



$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma' = R\,\sigma\,R^T$$

$$\sigma'_{11} = Y\cos^2\theta \qquad \sigma'_2 = -Y\sin\theta\cos\theta = -\frac{Y}{2}\sin 2\theta \quad \text{max at } 45^\circ$$

$$\rho \frac{D}{Dt}\vec{v} = \nabla_x \cdot \sigma^T + \rho \vec{b}$$

$$\rho \frac{Dv_i}{Dt} = \frac{\partial \sigma_{ji}}{\partial x_j} + \rho b_i$$

$$\boxed{d\vec{f} = \vec{t}\, ds}$$

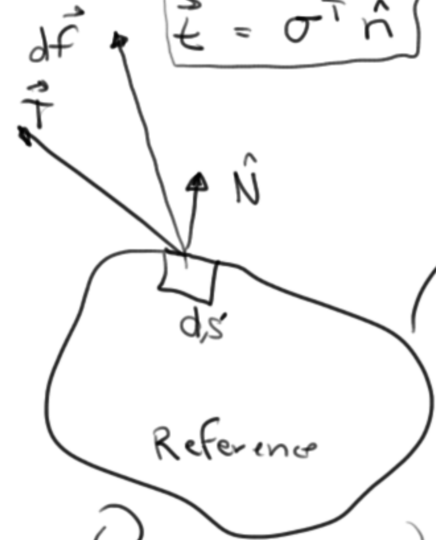
$$\rho \left[\frac{d\vec{v}}{dt} + \vec{v} \cdot \nabla_x \vec{v} \right] = \nabla \cdot \sigma^T + \rho \vec{b}$$

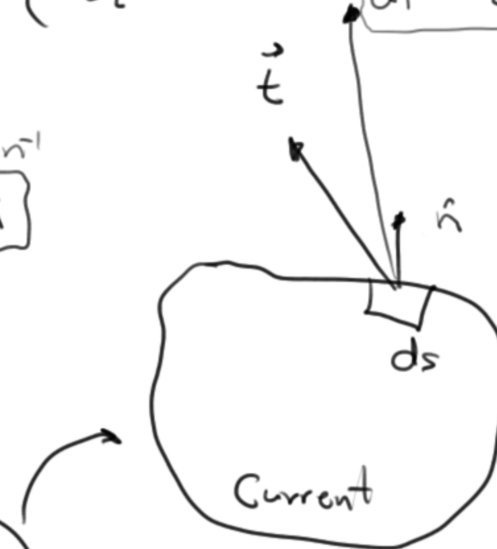
current

$$\boxed{\vec{t} = \sigma^T \hat{n}}$$

$$\rightarrow \quad \vec{t} = \sigma^T \hat{n}$$

$$\vec{T} = P^T \hat{N}$$





$$\boxed{d\vec{x} = F\, d\vec{X}}$$

$$dV = J\, dV_0$$

$$J = \det(F)$$

$$= \det\left(\frac{\partial x}{\partial X}\right)$$

$$\rho_0 \rightarrow$$

$$d\vec{f} = T\, dS'$$

$$\rightarrow \begin{cases} dV = d\vec{s}^{\,T} d\vec{x} \\ dV_0 = d\vec{S'}^{\,T} d\vec{X} \end{cases}$$

$$d\vec{s} = ds \cdot \hat{n}$$

$$d\vec{S'} = dS' \cdot \hat{N}$$

$$d\vec{s}^{\,T} \cdot d\vec{x} = J\, d\vec{S}^{\,T} d\vec{X}$$

$$d\vec{s}^{\,T} \cdot F \underbrace{\cancel{d\vec{X}}\, F^{-1}}_{I} = J\, d\vec{S'}^{\,T} \cdot \cancel{d\vec{X}}\, F^{-1}$$

$$d\vec{s}^{\,T} = J\, d\vec{S'}^{\,T} F^{-1}$$

$$\boxed{d\vec{s} = J\, F^{-T} d\vec{S'}}$$

Nanson's relation

$$df = \vec{t}\, ds = \sigma^T \hat{n}\, ds$$

$$df = \vec{T}\, ds' = P^T \hat{N}\, ds'$$

$$\vec{t} = \sigma^T \hat{n}$$

$$\vec{t} = \sigma \hat{n}$$

$$T = P \hat{N}$$

$$\sigma = \sigma^T$$

$$\sigma^T \underbrace{\hat{n}\, ds}_{d\vec{s}} = P^T \underbrace{\hat{N}\, ds'}_{d\vec{s}'}$$

$$\sigma^T d\vec{s} = P^T d\vec{s}'$$

$$J \sigma^T F^{-T} \cancel{d\vec{s}'} = P^T \cancel{d\vec{s}'}$$

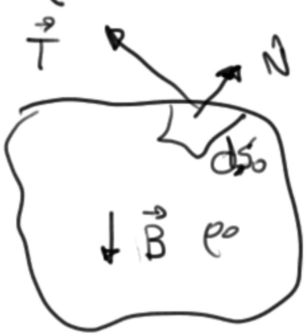
$$J \sigma^T F^{-T} = P^T$$

(Nominal stress) $\rightarrow \boxed{P = J F^{-1} \sigma}$

$$\frac{D\vec{v}}{Dt} = \frac{d\vec{v}}{dt} + \vec{v} \cdot \nabla_x \vec{v}$$

$\boxed{\sigma^{PK2} = P F^{-T}}$ $\Rightarrow$ 2nd Piola-Kirchoff stress

$\boxed{P^T \rightarrow \sigma^{PK1}}$ $\rightarrow$ (1st Piola-Kirchoff stress)



$$\int \underbrace{\vec{T}}_{P^T \hat{N}}\, ds' + \int \rho \vec{B}\, dV_0 = \frac{D}{Dt}\left[\int \rho_0 \vec{v}\, dV_0 \right]$$

$$\int \nabla \cdot P^T dV_0 + \int \rho \vec{B}\, dV_0 = \frac{D}{Dt}\left[\int \rho_0 \vec{v}\, dV_0 \right] = \int \rho_0 \frac{d\vec{v}}{dt}\, dV_0$$

$$\boxed{\rho_0 \frac{D\vec{v}}{Dt} = \nabla_X \cdot P^T + \rho_0 \vec{B}} \qquad \boxed{\rho_0 \frac{D\vec{v}}{Dt} = \nabla_X \cdot \sigma^{PK1} + \rho_0 \vec{B}}$$