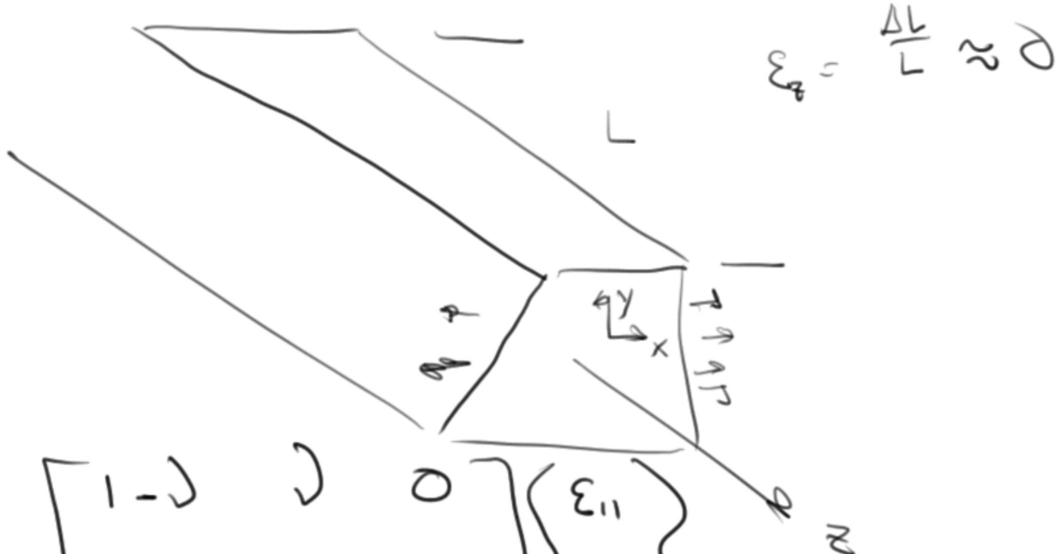
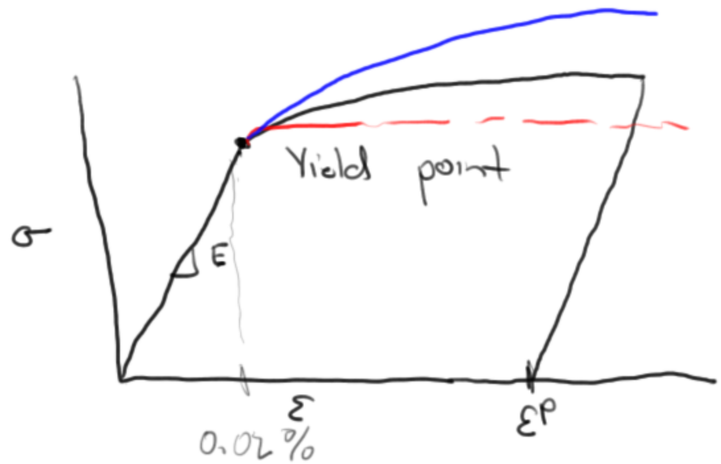
Plane strain

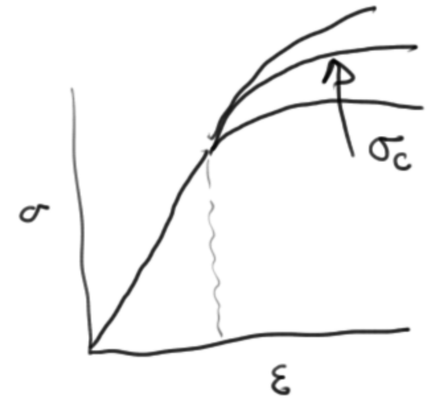
$\varepsilon_{33} = \varepsilon_{13} = \varepsilon_{23} = 0$

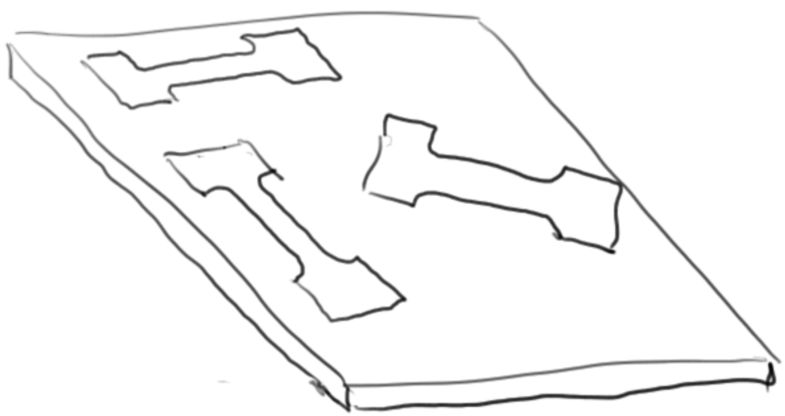


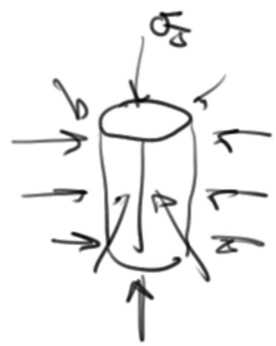
$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & 1-2\nu \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix}$$

$$\left\{ \vec{\varepsilon} \right\} = \frac{1+\nu}{E} \begin{bmatrix} 1-\nu & -\nu & 0 \\ -\nu & 1-\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \vec{\sigma} \right\}$$

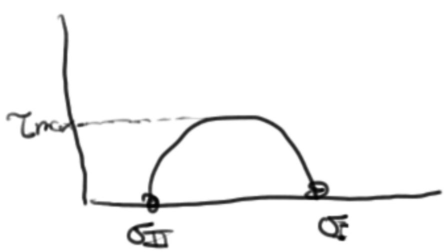




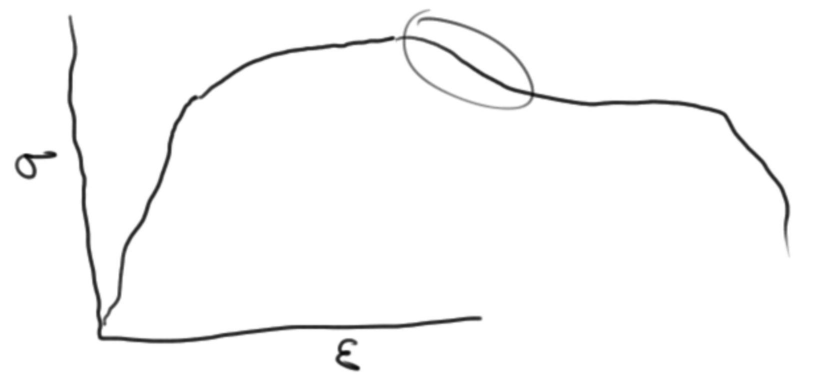


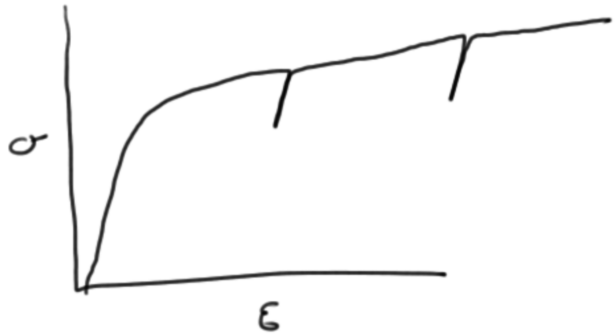


$$\sigma_{II} = \sigma_{III} = \sigma_c$$



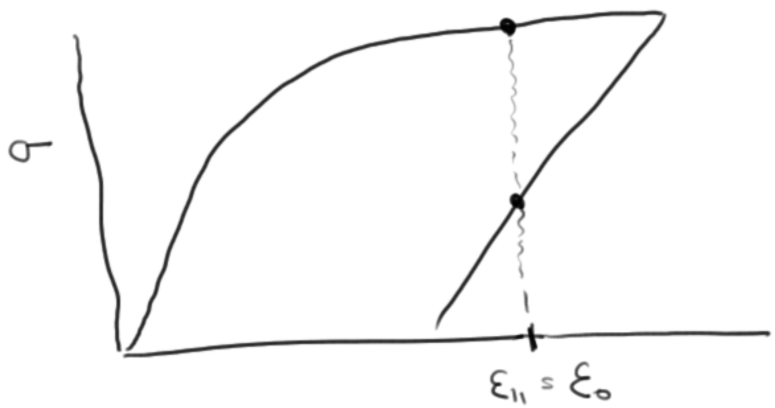
$$2\tau_{max} = \sigma_I - \sigma_{III}$$





$$\sigma_{ij} = \frac{\partial \omega}{\partial \varepsilon_{ij}} \qquad \omega(\rho\,\psi) = \omega(\varepsilon_{ij}, T)$$

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$\sigma = \sigma(\varepsilon_{ij}, T, \xi)$$

ξ may be "physical" variable

→ structure

→ physico-chemical reaction

→ phase changes

→ densities of structural defects

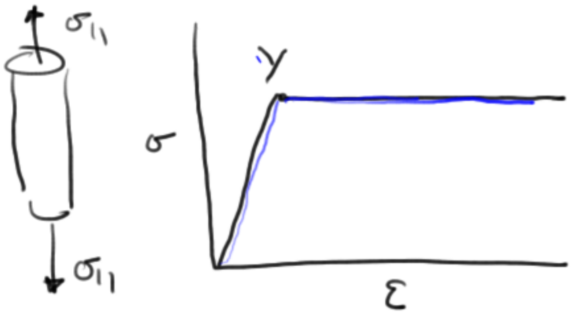
→ <u>phenomenological</u>

→ eg. plastic strain

$$\varepsilon = \varepsilon^e + \varepsilon^p$$ true for small strains $\|\nabla u\| \ll 1$

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$$

$$\dot{\varepsilon}^e = \dot{\varepsilon} - \dot{\varepsilon}^p$$



"Perfectly plastic"

$\sigma_{11} = 10$ MPa

$\sigma_{22} = 20$ MPa

$Y = 15$ MPa

Is the material yielding?

$$\sigma = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

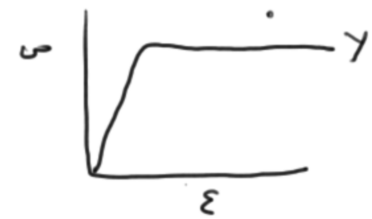
$\sigma_{11} = E\,\varepsilon_{11}$ (elastic)

$\sigma_{11} = Y$ (plastic)

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} Y/3 & 0 & 0 \\ 0 & Y/3 & 0 \\ 0 & 0 & Y/3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}Y & 0 & 0 \\ 0 & -\frac{1}{3}Y & 0 \\ 0 & 0 & -\frac{1}{3}Y \end{bmatrix} = S_{ij}$$

$$\sigma_{eq} \equiv \sqrt{3J_2} = \sqrt{\frac{3}{2}S_{ij}S_{ij}} = Y$$

$\sigma_{eq} \Rightarrow \bar{\sigma} = \sigma_{vm}$ von Mises stress



<u>von Mises Plasticity (J_2 plastity)</u>

Assumption: Under triaxial stress state, the material is yielding when $\sigma_{eq} \geq Y$

$$\sigma_{eq} = \sqrt{3J_2} = \left[\frac{1}{2} \left\{ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{22} - \sigma_{33})^2 \right\} + 3\sigma_{12}^2 + 3\sigma_{13}^2 + 3\sigma_{23}^2 \right]^{1/2}$$

$\sigma_{11} = 10$ MPa   9

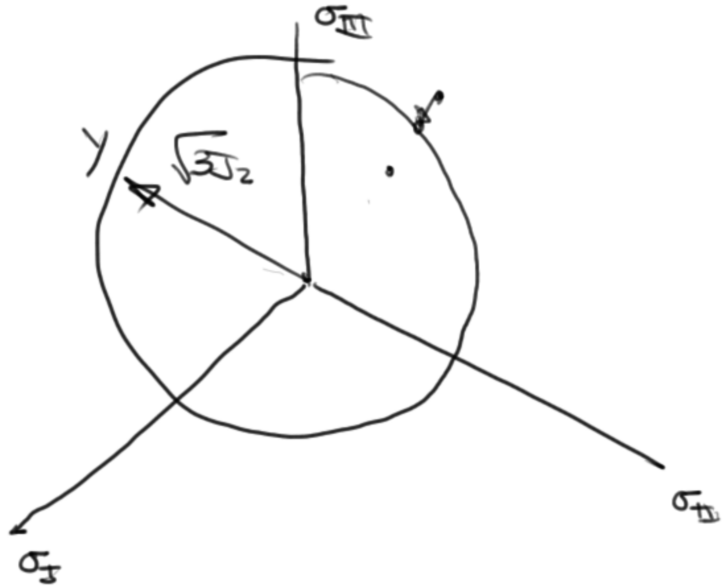
$\sigma_{22} = \sigma_{33} = 20$ MPa

$\sigma_{eq} = 10$ MPa $Y = 15$ $\therefore$ not yielding

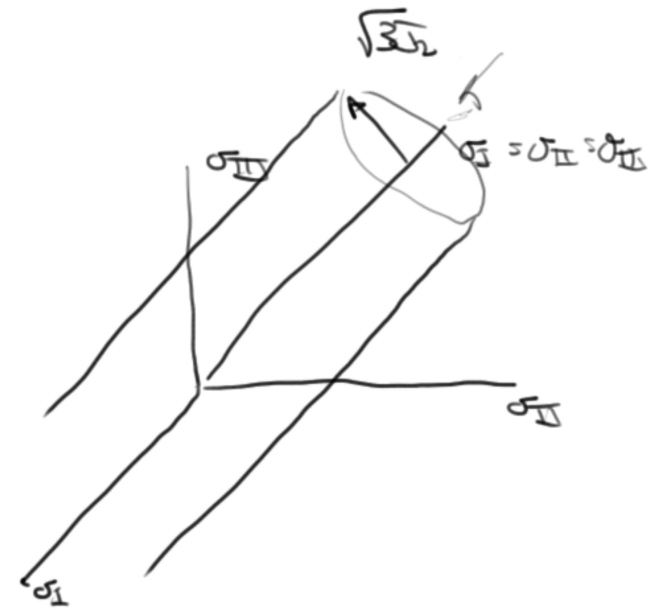
$f(\sigma_{ij}) = \sqrt{3J_2} - Y = 0$

$f(\sigma_{ij}) < 0 \Rightarrow$ elastic

$f(\sigma_{ij}) = 0 \Rightarrow$ plastic



σ_{III}
y
$\sqrt{3J_2}$
σ_I
σ_{II}



$\sqrt{3J_2}$
σ_{III}
$\sigma_I = \sigma_{II} = \sigma_{III}$
σ_{II}
σ_I