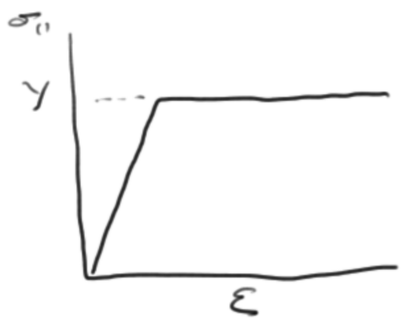


$$\varepsilon_{11} = 0.20\%$$

$$\varepsilon_{11}^e = \frac{\sigma_{11}}{E} - \frac{\nu}{E}\left(\cancelto{0}{\sigma_{22}} + \cancelto{0}{\sigma_{33}}\right) = \frac{\sigma_{11}}{E} = \frac{Y}{E}$$

$$\varepsilon_{22} = \frac{-\nu}{E}\sigma_{11} = \frac{-\nu Y}{E}$$

$$\varepsilon_{33} = \frac{-\nu}{E}\sigma_{11} = \frac{-\nu Y}{E}$$

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$$

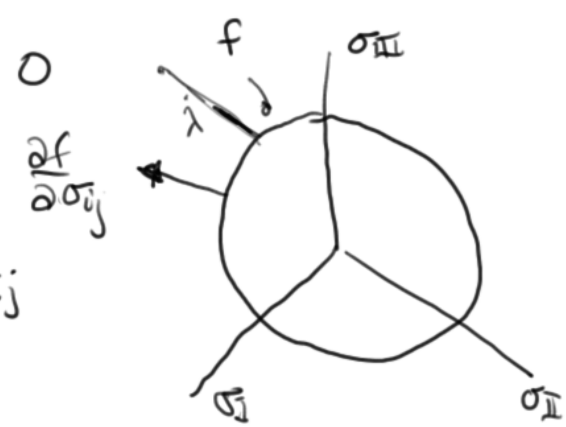
Flow rule

$$\boxed{\begin{aligned} \dot{\varepsilon}^p &= \dot{\lambda}\,\frac{\partial f}{\partial \sigma_{ij}} \\ d\varepsilon^p &= d\lambda\,\frac{\partial f}{\partial \sigma_{ij}} \\ d\varepsilon^p &= d\lambda\, S_{ij} \end{aligned}}$$

$$f = \sqrt{3J_2} - Y = 0$$

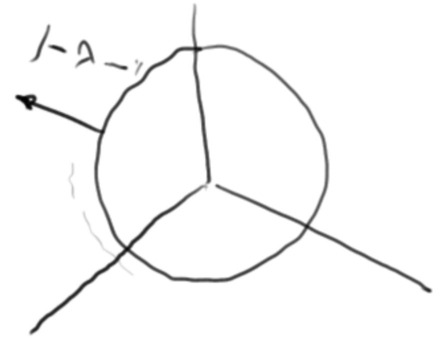
$$= J_2 - \frac{Y^2}{3} = 0$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}}(J_2) = S_{ij}$$



associated flow rule

$$\dot{\varepsilon}^p = \dot{\lambda}\,\frac{\partial f}{\partial \sigma_{ij}} \qquad Q_{ij}$$



$$\dot{\varepsilon}^p = \dot{\lambda}\,\frac{\frac{\partial f}{\partial \sigma_{ij}}}{\left|\frac{\partial f}{\partial \sigma_{ij}}\right|} = \dot{\lambda}\,\frac{S_{ij}}{|S_{ij}|} \qquad |S_{ij}| = \sqrt{S_{ij}S_{ij}}$$

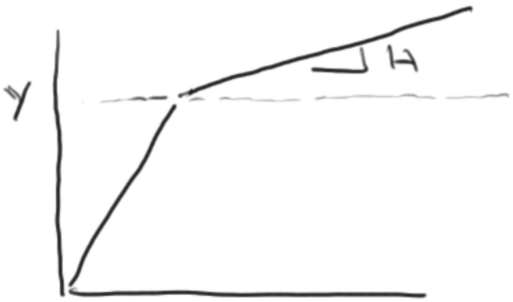
$$d\varepsilon^p = d\lambda \begin{bmatrix} \frac{2Y}{3} & 0 & 0 \\ 0 & -\frac{Y}{3} & 0 \\ 0 & 0 & -\frac{Y}{3} \end{bmatrix} \qquad \frac{2Y}{3} = \varepsilon_{11}^p, \quad -\frac{Y}{3} = -\frac{\varepsilon_{11}^p}{2}$$

$$\varepsilon_{11} = 0.2\%$$

$$\varepsilon_{11} = \frac{Y}{E} + \varepsilon_{11}^p \Rightarrow \varepsilon_{11}^p = \varepsilon_{11} - \frac{Y}{E}$$

$$\varepsilon_{22} = -\frac{\nu}{E}Y - \frac{1}{2}\left(\varepsilon_{11}^p\right) = -\frac{\nu}{E}Y - \frac{1}{2}\left(\varepsilon_{11} - \frac{Y}{E}\right)$$

like ε_{33}



$$f = \sqrt{3J_2} - Y(\varepsilon^p) = 0$$

$$= \sqrt{3J_2} - Y - H\varepsilon^p = 0$$

equivalent plastic strain-rate $\quad \dot{\varepsilon}^p = \sqrt{\frac{2}{3}\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij}} = \sqrt{\frac{2}{3}}\,\dot{\lambda}$

$$\varepsilon^p = \int_0^{t_f} \dot{\varepsilon}^p\,dt$$

Flow $\Rightarrow \quad \dot{\varepsilon}^p_{ij} = \dot{\lambda}\overbrace{\left(\frac{S_{ij}}{|S_{ij}|}\right)}^{Q_{ij}} = \dot{\lambda}\,Q_{ij}$

$$\varepsilon^d_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}$$

$$\dot{\varepsilon}^d_{ij} = \dot{\varepsilon}^{de}_{ij} + \dot{\varepsilon}^{dp}_{ij} \Rightarrow \dot{\varepsilon}^d_{ij} - \dot{\varepsilon}^{de}_{ij} - \dot{\varepsilon}^{dp}_{ij} = 0$$

$$\underline{\dot{\varepsilon}_{ij}Q_{ij} - \dot{\varepsilon}^{de}_{ij}Q_{ij} - \dot{\varepsilon}^{dp}_{ij}Q_{ij} = 0}$$

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

$$\varepsilon^{d}_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij}$$

$$\varepsilon_{ij} = \varepsilon^{d}_{ij} + \frac{1}{3} \varepsilon_{kk} \delta_{ij}$$

$$= \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \left(\varepsilon^{d}_{ij} + \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right)$$

$$= 2\mu \varepsilon^{d}_{ij} + \varepsilon_{kk} \delta_{ij} \underbrace{\left(\lambda + \frac{2}{3} \mu \right)}_{K}$$

$$\boxed{\sigma_{ij} = 2\mu \varepsilon^{d}_{ij} + \varepsilon_{kk} \delta_{ij} K} \Rightarrow \sigma_{ii} = 2\mu \overset{0}{\cancel{\varepsilon^{d}_{ii}}} + K \varepsilon_{kk} \overset{3}{\cancel{\delta_{ii}}} = \underbrace{3 K \varepsilon_{kk}}$$

$$s_{ij} = \boxed{\sigma_{ij}} - \underbrace{\frac{1}{3} \sigma_{kk} \delta_{ij}}$$

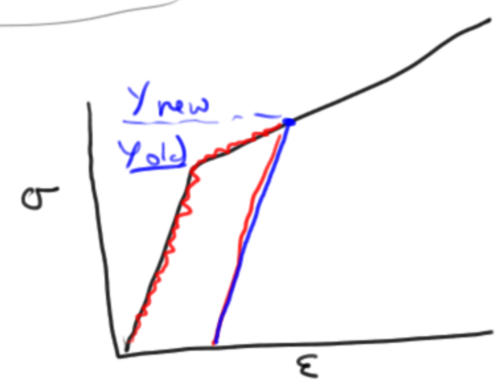
$$s_{ij} = 2\mu \varepsilon^{d}_{ij} + \cancel{K \varepsilon_{kk} \delta_{ij}} + \cancel{\frac{1}{3} (3 K \varepsilon_{kk}) \delta_{ij}}$$

$$\boxed{s^{e}_{ij} = 2\mu \varepsilon^{de}_{ij}}$$

$$0 = \dot{\varepsilon}^{d}_{ij} Q_{ij} - \dot{\varepsilon}^{de}_{ij} Q_{ij} - \dot{\varepsilon}^{dp}_{ij} Q_{ij}$$

$$\dot{\varepsilon}^{dp} = \dot{\lambda} Q_{ij}$$

$$\dot{S}_{ij} = 2\mu\, \dot{\varepsilon}^{de}_{ij} \Rightarrow \dot{\varepsilon}^{de}_{ij} = \frac{\dot{S}_{ij}}{2\mu}$$



$$0 = \dot{\varepsilon}^{d}_{ij} Q_{ij} - \frac{\dot{S}_{ij}}{2\mu} Q_{ij} - \dot{\lambda}$$

$\dot{s}$

$$\boxed{\dot{\varepsilon}^{d}_{ij} Q_{ij} - \frac{\dot{s}}{2\mu} - \dot{\lambda} = 0} \Rightarrow \bigstar$$

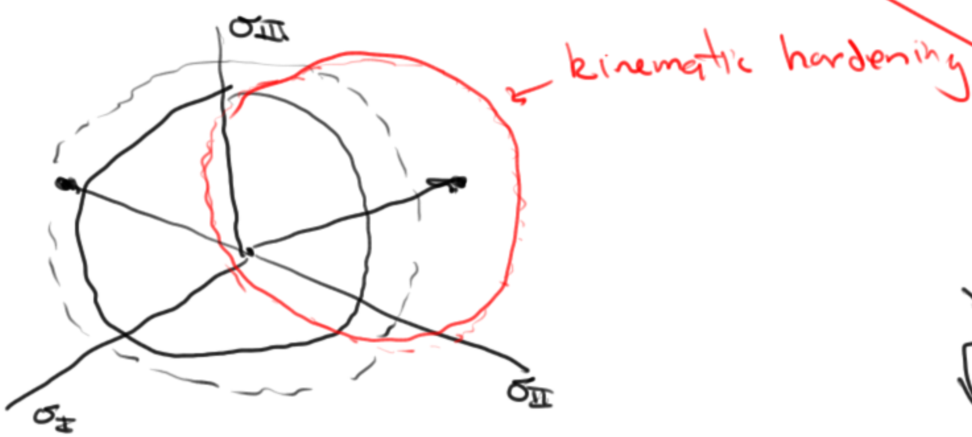
$$\boxed{f = \sqrt{3J_2} - Y - H\,\varepsilon^p}$$

Ex Isotropic hardening

Kuhn-Tucker constrait eqns.

$$f \leq 0 \;,\quad \dot{\lambda} \geq 0 \;,\quad \dot{\lambda} f = 0$$

$$\boxed{\dot{f} = 0}$$



$$\sqrt{\tfrac{3}{2}}\, \dot{s} - H \dot{\varepsilon}^p = 0$$

$$\sqrt{\tfrac{3}{2}}\, \dot{s} - H \sqrt{\tfrac{2}{3}}\, \dot{\lambda} = 0 \Rightarrow \boxed{\dot{s} = \tfrac{2}{3} H \dot{\lambda}}$$

$$\dot{\lambda} = \varepsilon^{d}_{ij}\, Q_{ij} \left(\frac{H}{3\mu} + 1 \right)^{-1}$$

$$\underbrace{\dot{\varepsilon}^{de}_{ij}} = \dot{\varepsilon}^{d}_{ij} - \underbrace{\dot{\varepsilon}^{dp}}$$

$$= \dot{\varepsilon}^{d}_{ij} - \dot{\lambda}\, Q_{ij} = \underbrace{\dot{\varepsilon}^{d}_{ij} \left(1 - \left(\frac{H}{3\mu} + 1 \right)^{-1} \right)} \qquad \text{if} \quad f = 0$$

$$\dot{\sigma}_{ij} = \begin{cases} \dfrac{2H\mu}{H + 3\mu}\, \dot{\varepsilon}^{d}_{ij} + K\, \dot{\varepsilon}_{kk}\, \delta_{ij} & \text{if} \quad f = 0 \\ 2\mu\, \dot{\varepsilon}^{d}_{ij} + K\, \dot{\varepsilon}_{kk}\, \delta_{ij} & \text{if} \quad f < 0 \end{cases}$$