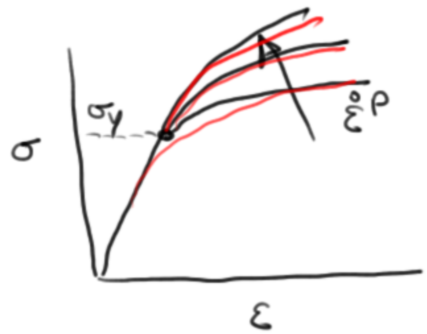
Viscoplastic



$$f = \sqrt{\tfrac{3}{2}S} - \sigma_y\left(1 + \beta\,\dot{\varepsilon}^p\right)^N$$

$$\dot{f} = 0$$

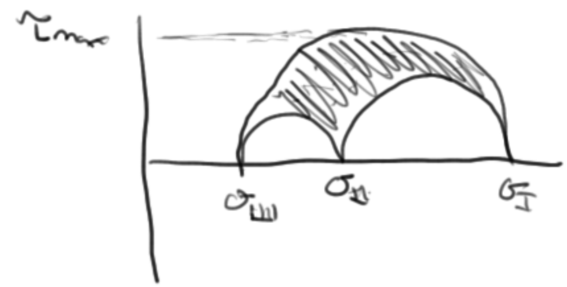
$$0 = \sqrt{\tfrac{3}{2}}\,\dot{S} - N\sigma_y\beta\,\ddot{\varepsilon}^p\left(1 + \beta\,\dot{\varepsilon}^p\right)^{N-1} \qquad \dot{\varepsilon}^p = \dot{\lambda} S_{ij}$$

$$= \sqrt{\tfrac{3}{2}}\,\dot{S} - \sqrt{\tfrac{2}{3}}\,N\sigma_y\beta\,\ddot{\lambda}\left(1 + \beta\sqrt{\tfrac{2}{3}}\,\dot{\lambda}\right)^{N-1}$$

$$\dot{S} = \tfrac{2}{3}N\sigma_y\beta\,\ddot{\lambda}\left(1 + \beta\sqrt{\tfrac{2}{3}}\,\dot{\lambda}\right)^{N-1} \Rightarrow \bigstar$$

$$0 = \dot{\varepsilon}^d_{ij}Q_{ij} - \frac{1}{3M}N\sigma_y\ddot{\lambda}\left(1 + \beta\sqrt{\tfrac{2}{3}}\,\dot{\lambda}\right)^{N-1} - \dot{\lambda}$$

$$f(J_2) = \sqrt{3J_2} - Y(\varepsilon^p, \dot{\varepsilon}^p)$$



Other yield surfaces

Maximum shear stress (Tresca Criterion)

(Lubliner)

$$f = \frac{1}{2} \max \left\{ |\sigma_I - \sigma_{II}|, |\sigma_{II} - \sigma_{III}|, |\sigma_I - \sigma_{III}| \right\}$$

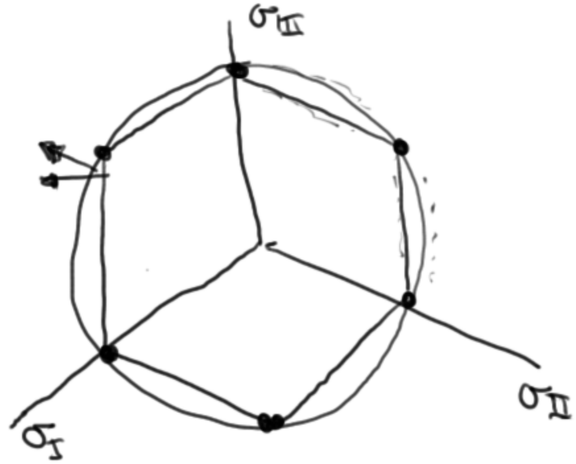
$$= 4J_2^3 - 27J_3^2 - 9J_2^2 + Y - 6J_2Y^4 - Y^6 = 0$$

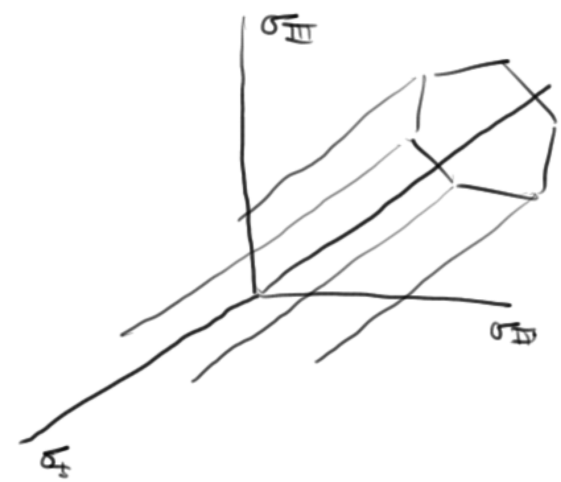
$$J_3 = \det(s_{ij}) = \sigma_I^d \sigma_{II}^d \sigma_{III}^d$$

$$s_{ij} = \begin{bmatrix} -\frac{2}{3}Y & 0 & 0 \\ 0 & +\frac{1}{3}Y & 0 \\ 0 & 0 & +\frac{1}{3}Y \end{bmatrix}$$

$J_3 = \frac{2}{27} Y^3$ tension

$J_3 = -\frac{2}{27} Y^3$ compression





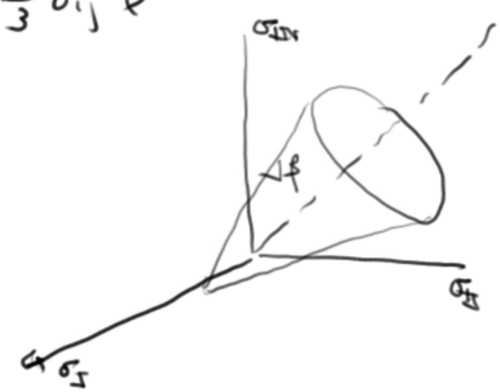
Drucker - Prager

$$f(p, J_2) = \sqrt{3J_2} - \beta p - Y(\varepsilon^p, \dot{\varepsilon}^p, T) = 0$$

pressure dep. in the plastic flow rule

$$p = -\frac{1}{3}\sigma_{kk}$$

$$\frac{\partial f}{\partial \sigma_{ij}} \Rightarrow \beta \frac{\partial p}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}}\left(-\frac{1}{3}\sigma_{kk}\right)\beta = -\frac{1}{3}\delta_{ik}\delta_{jk}\beta = -\frac{1}{3}\delta_{ij}\beta$$



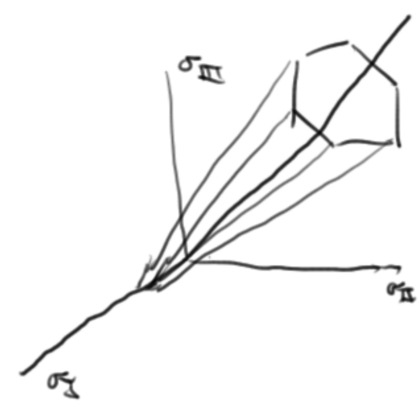
Mohr - Coulomb

$$\sigma_I - \sigma_{III} + (\sigma_I + \sigma_{III})\sin(\phi) = Y\cos(\phi)$$

ϕ angle-of-internal friction

In terms of the invariants

$$\frac{1}{3} I_1 \sin\phi + \sqrt{J_2}\left\{\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\sin\phi\right\} = \frac{Y}{2}\cos(\phi)$$

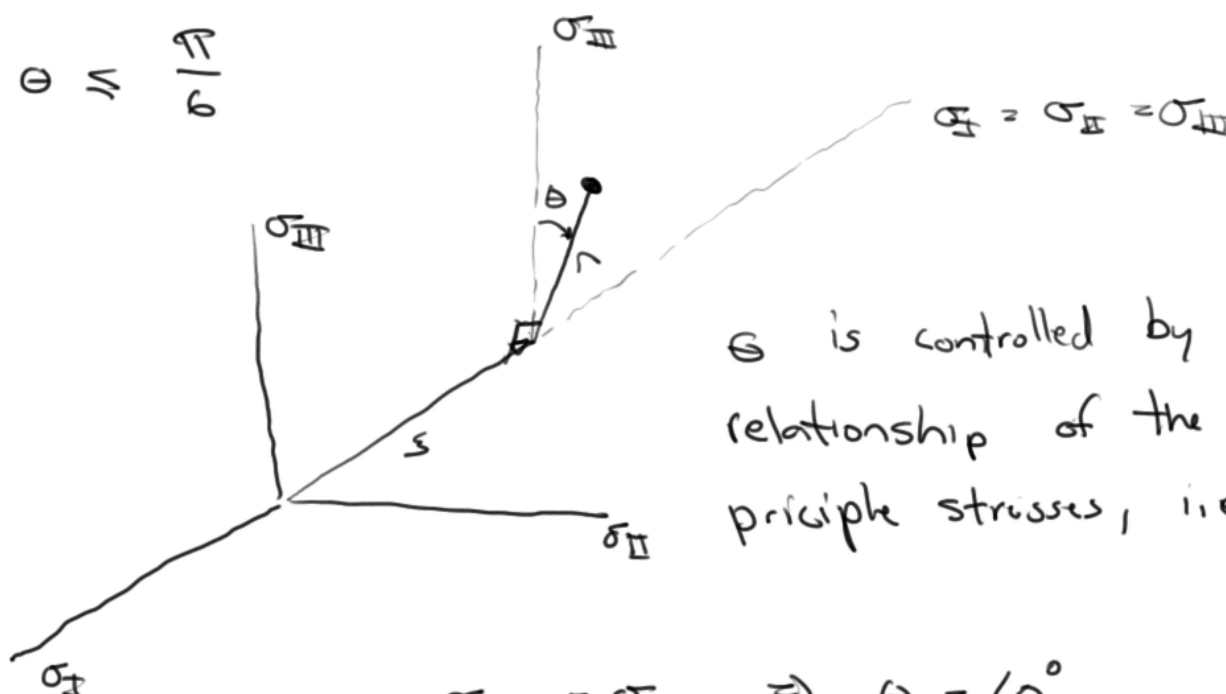


$\theta \rightarrow$ Lode angle

Lode angle

$$\theta = \frac{1}{3} \sin^{-1} \left(\frac{3\sqrt{3}}{2} \quad \frac{J_3}{J_2^{3/2}} \right)$$

$$-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}$$



θ is controlled by the relationship of the intermediate priciple stresses, i.e σ_I to σ_{II}, σ_{III}

when $\sigma_{II} = \sigma_{III} \Rightarrow \theta = 60^\circ$

$\sigma_{II} = \sigma_I \Rightarrow \theta = 0^\circ$