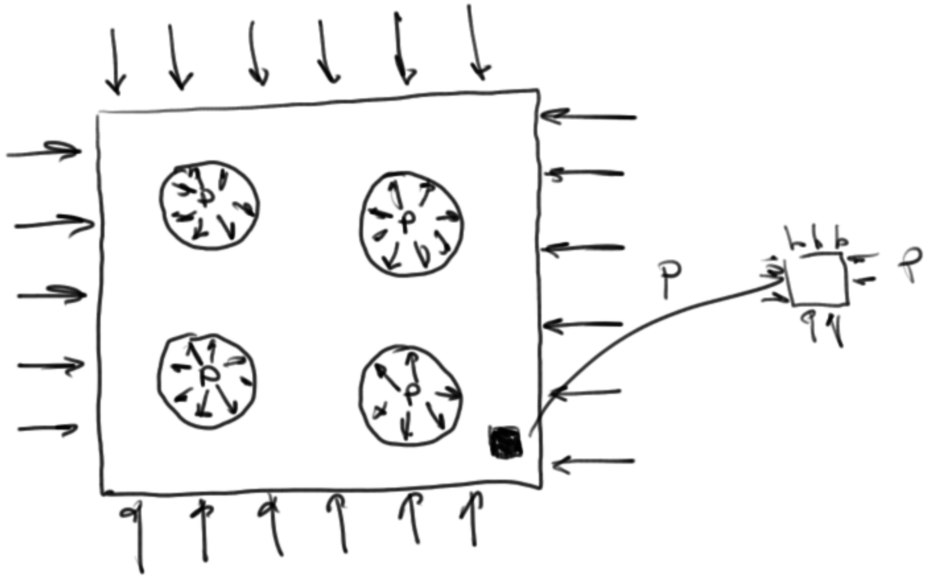
$$\sigma_{ij} = S_{ij} + \frac{1}{3}\sigma_{kk}\,\delta_{ij} = S_{ij} - p\,\delta_{ij}$$

$$p = -\frac{1}{3}\sigma_{kk}$$



$$\sigma_{ij} = -p\,\delta_{ij} = 2\mu\,\cancel{\varepsilon^d_{ij}}^{\,0} + K_s\,\varepsilon_{kk}\,\delta_{ij}$$

$$\varepsilon_{kk} = \frac{-p}{K_s}$$

$$\varepsilon_{ij} = \cancel{\varepsilon^d_{ij}}^{\,0} + \frac{1}{3}\varepsilon_{kk}\,\delta_{ij}$$

$$\boxed{\varepsilon_{ij} = \frac{-p}{3K_s}\,\delta_{ij}}$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \Rightarrow \varepsilon_{kl} = D_{klij}\,\sigma_{ij}$$

where $D = C^{-1}$ or $C_{ijkl}\,D_{mnop} = \underbrace{\delta_{im}\,\delta_{jn}\,\delta_{ko}\,\delta_{lp}}_{\text{4th-order identity tensor}}$

an arbitrary σ + p

$$\varepsilon_{kl} = \underbrace{D_{klij}}_{C^{-1}} \left(\sigma_{ij} + p\,\delta_{ij} \right) - \frac{1}{3K_s}\, p\, \delta_{kl}$$

effective stress

$$\sigma' = \sigma^{s}_{ij} + \alpha\, p\, \delta_{ij} = C_{ijkl}\,\varepsilon_{kl} = \underbrace{C_{ijkl} \Big[D_{klij}}_{\delta_{ik}\delta_{jl}\delta_{ki}\delta_{lj}\sigma_{ij}} \left(\sigma^{s}_{ij} + p\,\delta_{ij} \right) - \frac{1}{3K_s}\, p\, \delta_{kl} \Big]$$

$$\downarrow$$

$$\cancel{\sigma^{s}_{ij}} + \alpha\, \cancel{p}\, \frac{\delta_{ij}\cancelto{3}{\delta_{ij}}}{3} = \underbrace{\cancel{\sigma^{s}_{ij}}} + \cancel{p}\, \frac{\delta_{ij}\cancelto{3}{\delta_{ij}}}{3} - \frac{\delta_{ij}\, C_{ijkl}}{3 \cdot 3K_s}\, \cancel{p}\, \delta_{kl}$$

$$\boxed{\alpha = 1 + \frac{\delta_{ij}\, C_{ijkl}\, \delta_{kl}}{9K_s}} \leftarrow \text{Biot's coefficient}$$

$$\frac{\delta_{ij} C_{ijkl} \delta_{kl}}{9} = \frac{9\lambda + 6\mu}{9} = K_T$$

$$\alpha = 1 - \frac{K_T}{K_s}$$

$K_s >> K_T \qquad \alpha \to 1 \qquad$ sands

rocks + concrete $\quad \alpha \approx \frac{2}{3} \qquad 0.5 < \alpha \quad 0.8$

$$\boxed{\sigma' = \sigma_{ij} + \alpha p \delta_{ij}}$$

Assumptions

1. Isothermal
2. No mass exchange between solid + fluid
3. Low Reynold's #, convection negligible + Darcy applies
4. Inertial effects are negligible

$$\frac{D\vec{v}^s}{Dt} = 0$$

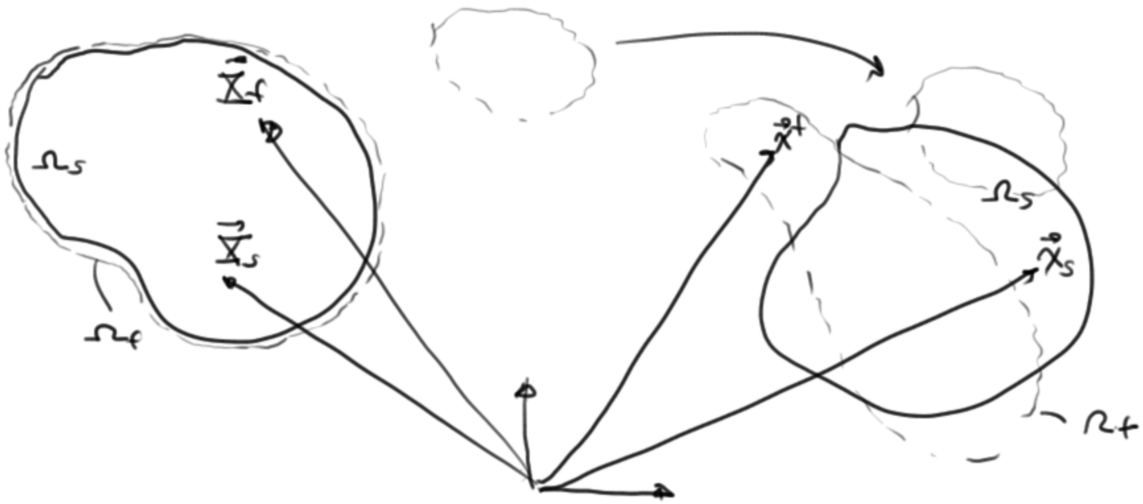
Total density

$$\rho = \rho^s + \rho^f = \phi^s \rho_s + \phi^f \rho_f \qquad \phi^s + \phi^f = 1$$

ρ^γ is intrinsic density of γ constituents

$$\rho^\gamma = \frac{m_\gamma}{V_T}$$

$$\rho_\gamma = \frac{m_\gamma}{V_\gamma}$$



Problem is formulate on the trajectory of the solid so

$$\vec{x}^s = \vec{x}^s(\vec{X}^s, t) = \vec{x}(\vec{X}, t)$$

Total Cauchy stress is

$\sigma^s \rightarrow$ partial stress defined on volume of solid grains, V^s

$\sigma^f \rightarrow$ " " " " " of pores, V^f

$$V = V^s + V^f$$

fluid phase Cauchy stress is isotropic (no shear resitance)

$$p^f I = -\sigma^f \quad , \quad p^f = -\frac{1}{3}\sigma^f_{kk} \quad , \quad p^f = -\frac{1}{3\phi^f}\, tr(\phi^f \sigma_f)$$

$$\sigma^s = \sigma' + \alpha p^f I$$

$$\sigma = \sigma' - \alpha p^f I$$

$$J = \det(F)$$

<u>Recall</u>

$$P = J F^{-1}\sigma$$

$$P^T = \sigma^{1PK} = (J F^{-1}\sigma)^T = J \sigma^T F^{-T}$$ ← Piola - Transformation

$$\sigma^{PK1} = (\sigma^{1PK})' - J \alpha p^f F^{-T}$$

$$\nabla_{\underline{X}} \cdot (\sigma^{1PK})^s + \rho^s \vec{b} + H^s = 0$$

$$\nabla_{\underline{X}} \cdot (\sigma^{1PK})^f + \rho^f \vec{b} + H^f = 0$$

H^f & H^s are interactive body forces per unit reference volume exerted on the coorresponding phase due to drag, lift, virtual mass effect, history effects, and relative spin that balance internally

$$H^f + H^s = 0$$

$$\nabla_{\underline{X}} \cdot (\sigma^{PK1}) + (\rho^s + \rho^f) \vec{b} = \vec{0}$$

balance of linear momentum