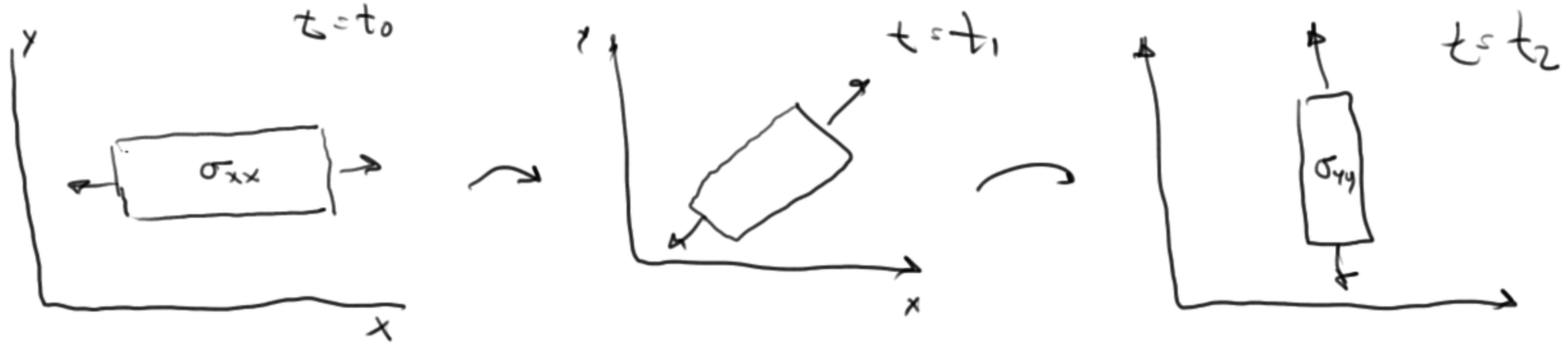
$\dot{\sigma} = f(\dot{\varepsilon})$ $\qquad\qquad$ $\sigma = R\sigma R^T$

$$\dot{\sigma} = \dot{R}\sigma R^T + R\dot{\sigma}R^T + R\sigma\dot{R}^T \neq R\dot{\sigma}R^T$$



$$\overset{\nabla}{\sigma} = \dot{\sigma} + \sigma\Omega - \Omega\sigma \qquad\qquad L = D + W$$

$\Omega = W$ ← spin-tensor ⇒ Jaumann objective rate-of Cauchy stress

$\Omega = R$ from $F = RU = VR$

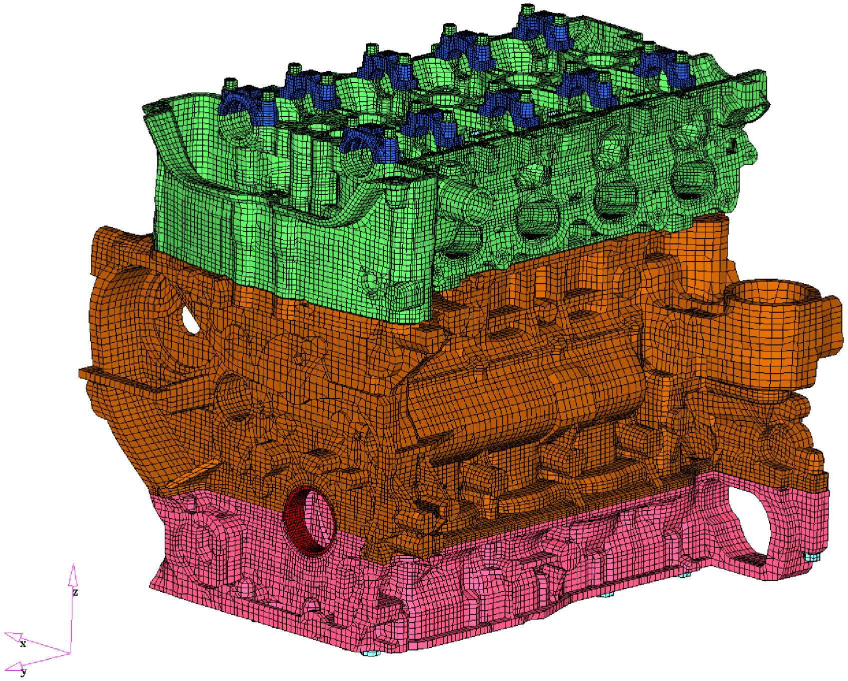
↳ Green-Naghdi 〃 corotational rate-of-Cauchy stress

$$\overset{\nabla}{\sigma} = R\overset{\nabla}{\sigma}R^T$$

<u>Truesdell</u>

$$\overset{\nabla}{\sigma} = \dot{\sigma} - L\sigma - \sigma L^T + \mathrm{tr}(L)\sigma$$

Introduction to Finite Element Analysis



“The purpose of computing is insight, not numbers.”

--Book Dedication: RW Hamming (1971).
Introduction to Applied Numerical
Analysis. McGraw Hill.

"The purpose of analysis is insight, not numbers."

What is analysis?

- From the Greek word *analyein*, meaning *“to break up”*
- An informal definition in the context of science and engineering would be *“probing into, or simulating nature”*

Why do analysis?

- Analysis is the key to effective design

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?
    - One that works!

http://www.youtube.com/watch?v=_ve4M4UsJQo

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?
    - ~~One that works!~~

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?
    - One that performs the task efficiently

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?
    - One that performs the task efficiently
    - Economical

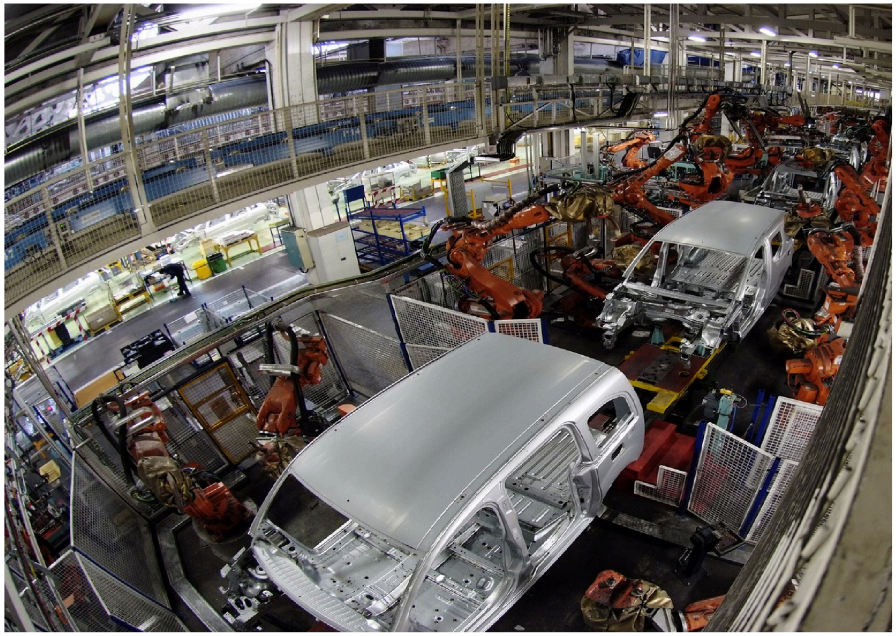
VS

Why do analysis?

- Analysis is the key to effective design
  - What is an effective design?
    - One that performs the task efficiently
    - Economical
    - Safe

Why do analysis?

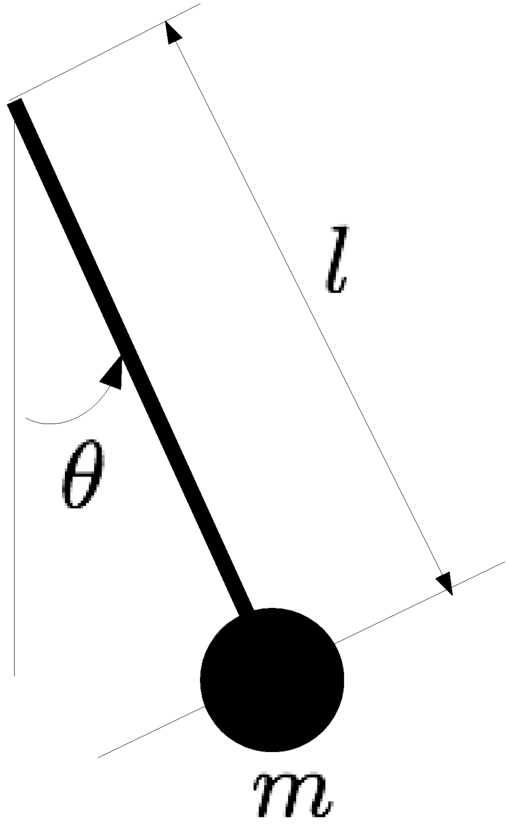
- Analysis is the key to effective design
  - What is an effective design?
    - One that performs the task efficiently
    - Economical
    - Safe
    - Manufacturable



Why do analysis?

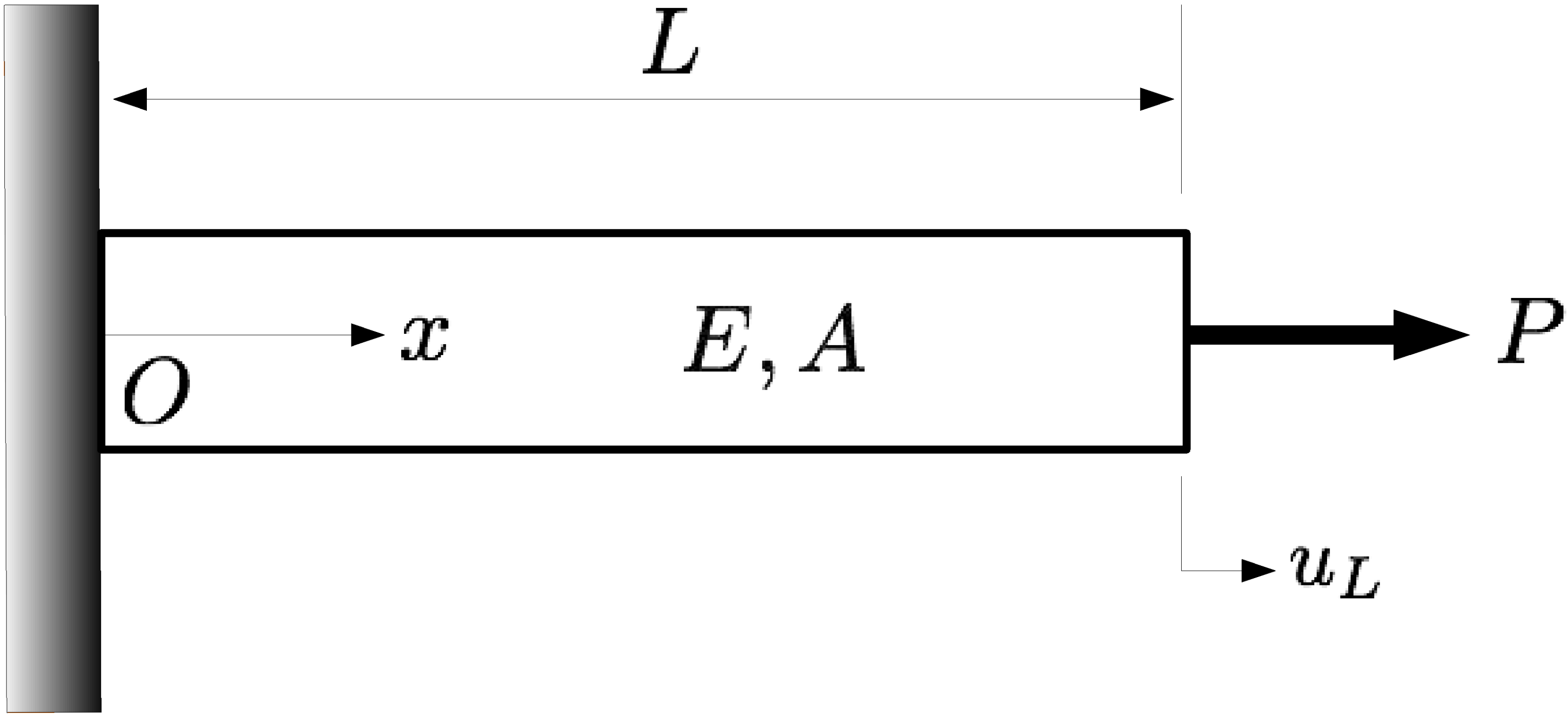
- Analysis is the key to effective design
  - What is an effective design?
    - One that performs the task efficiently
    - Economical
    - Safe
    - Manufacturable
    - Appealing

Analysis is performed by utilizing mathematical models



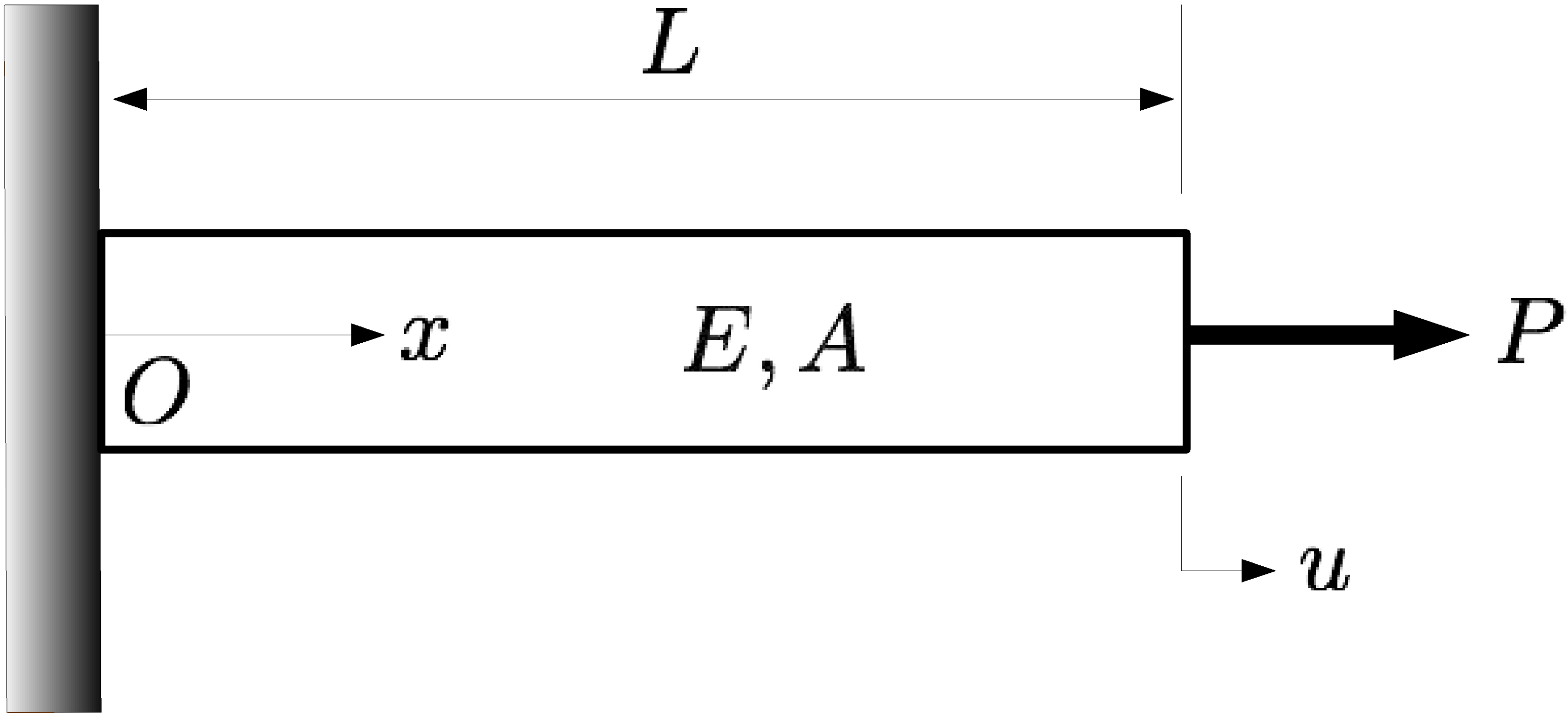
$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

An example from solid mechanics

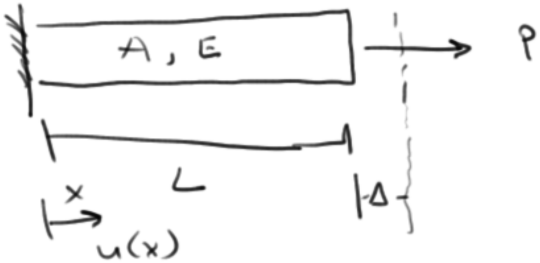


What is the equation of motion in terms of displacement u ?

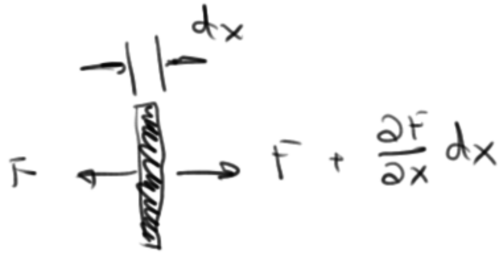
An example from solid mechanics



To the whiteboard...



$$\Delta = \frac{PL}{AE}$$



$$-\cancel{F} + \cancel{F} + \frac{\partial F}{\partial x}\cancel{dx} = \rho A(x)\cancel{dx}\cancelto{0}{\frac{\partial^2 u}{\partial t^2}}$$

$$\frac{\partial}{\partial x}(F) = 0$$

$$F = \sigma A(x)$$

$$= E\varepsilon A(x)$$

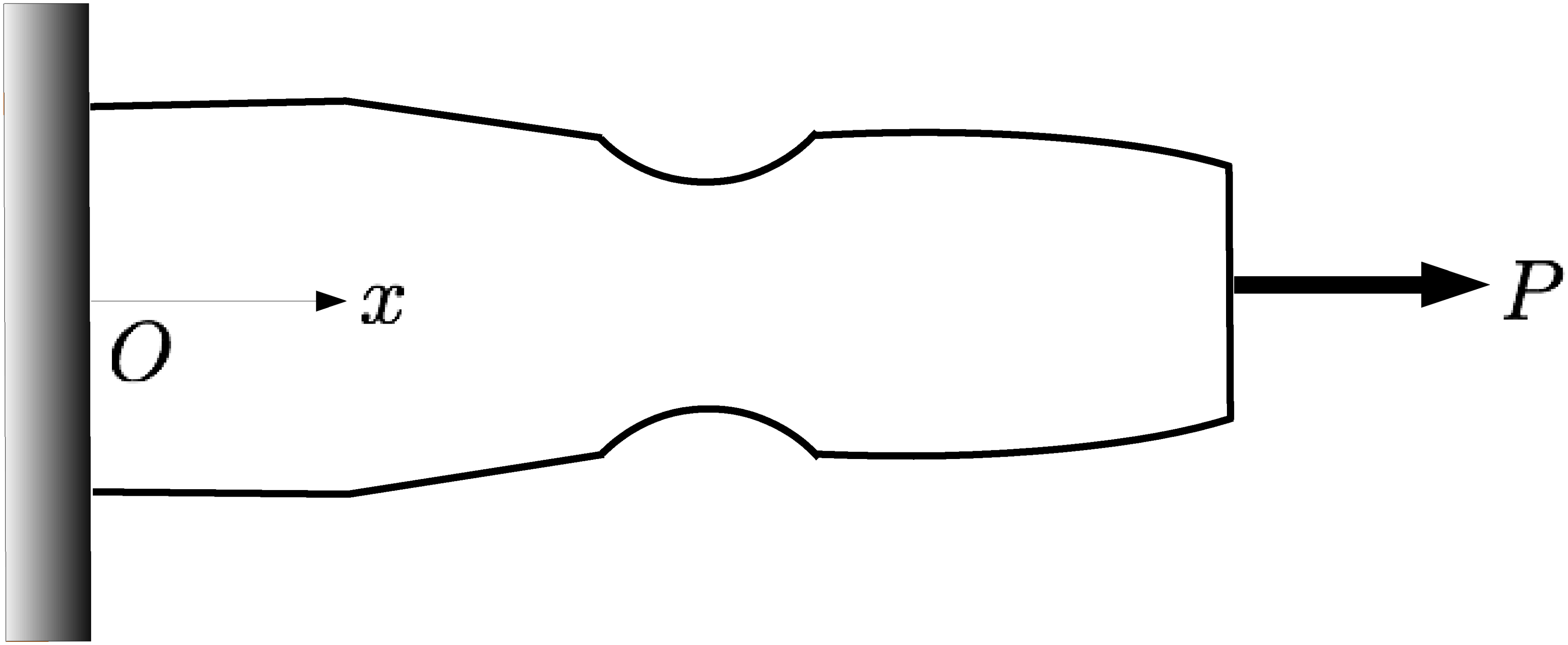
$$= E\frac{\partial u}{\partial x} A(x)$$

$$\boxed{\frac{\partial}{\partial x}\left[E A(x)\frac{\partial u}{\partial x}\right] = 0}$$

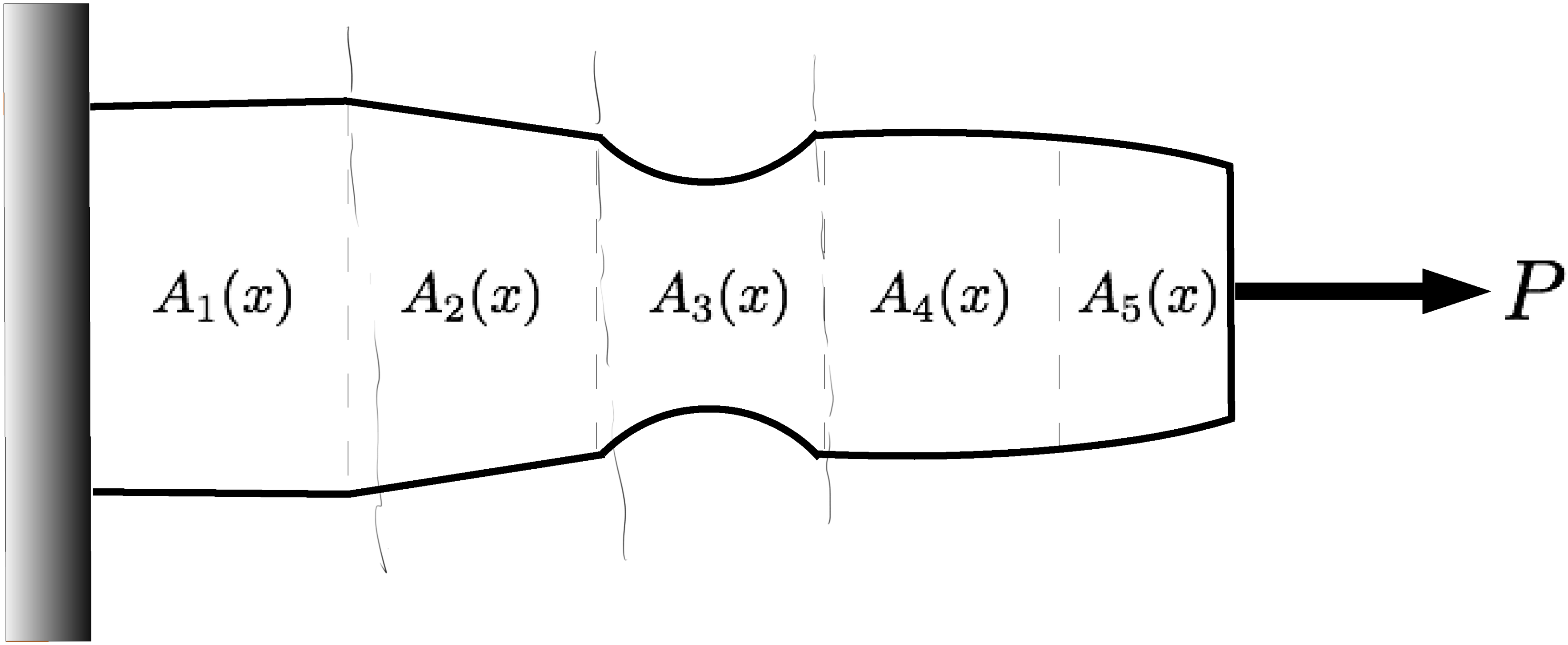
$$\boxed{u(0) = 0}$$

$$\boxed{E A(L)\frac{\partial u(L)}{\partial x} = P}$$

What if A is nonuniform?

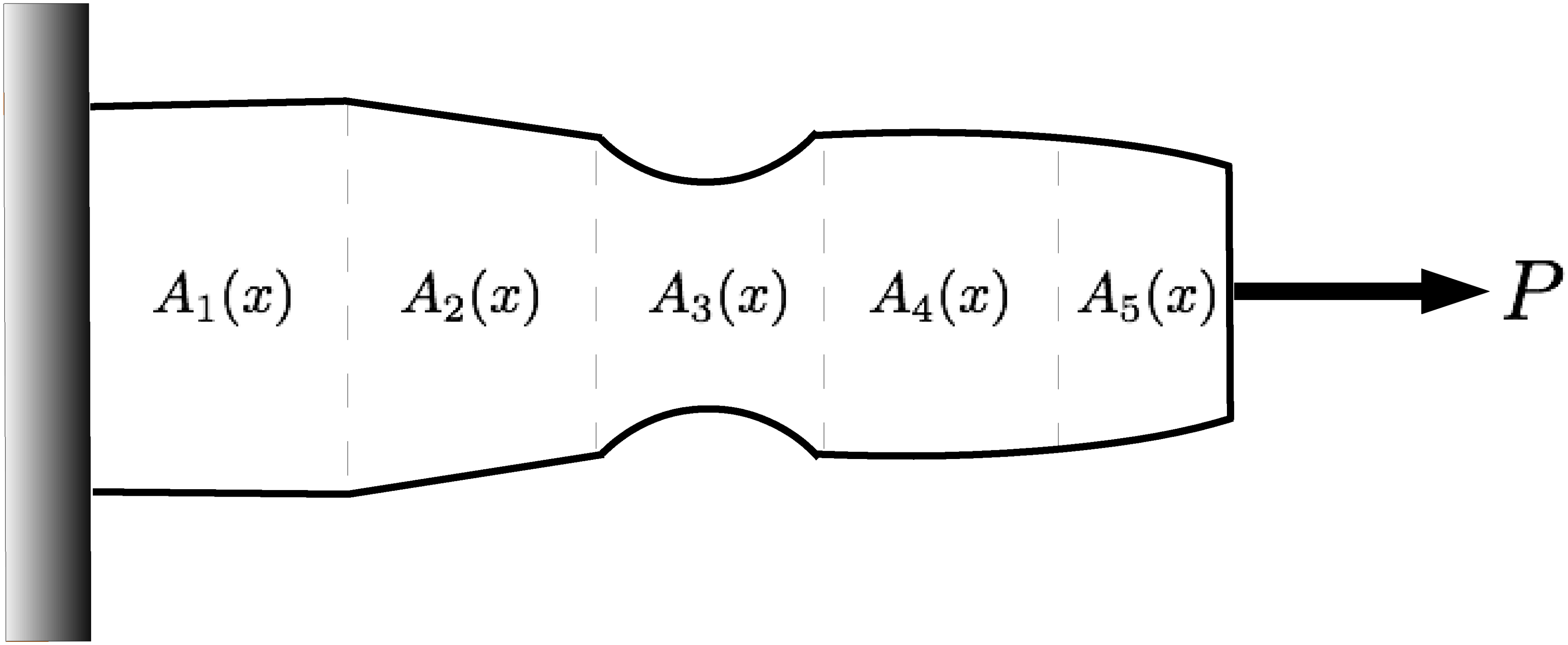


What if A is nonuniform?



We *discretize* the domain.

What if A is nonuniform?



Does the shape of each subdomain look familiar?

The Finite Element Method (FEM) in a nutshell

- The domain of the problem is represented by a collection of simple subdomains, called *finite elements.*
  - The collection of finite elements is called the *finite element mesh.*
- Over each finite element, the physical process is approximated by functions (polynomials or otherwise) and algebraic equations relating physical quantities at selective points, called *nodes,* are developed.
- The element equations are *assembled* using continuity and/or “balance” of physical quantities and solved.

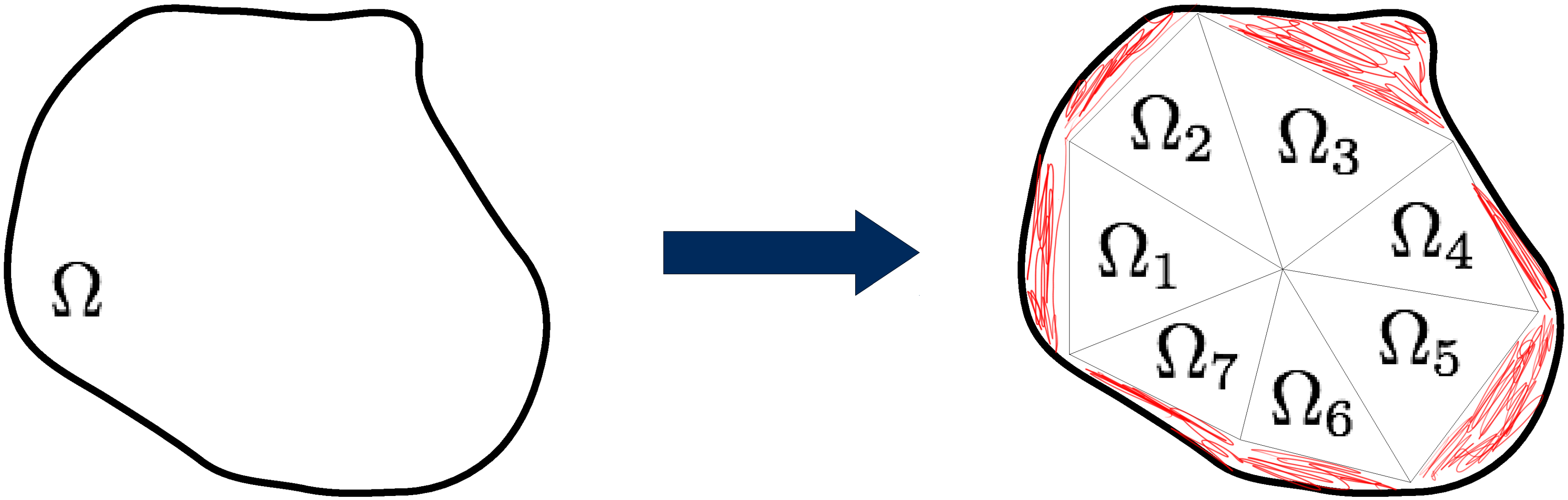
Notice I said the physical processes are *approximated* *over an element.*

- In the axial deformation problem posed earlier we solved the differential equations exactly.
- This is typically neither feasible nor efficient.
- In FEM we seek an approximation over the element of the form:

$$u \approx u_h = \sum_{j=1}^{n} u_j \psi_j + \underbrace{\sum_{j=1}^{m} c_j \phi_j}_{\text{XFEM /GFEM}}$$

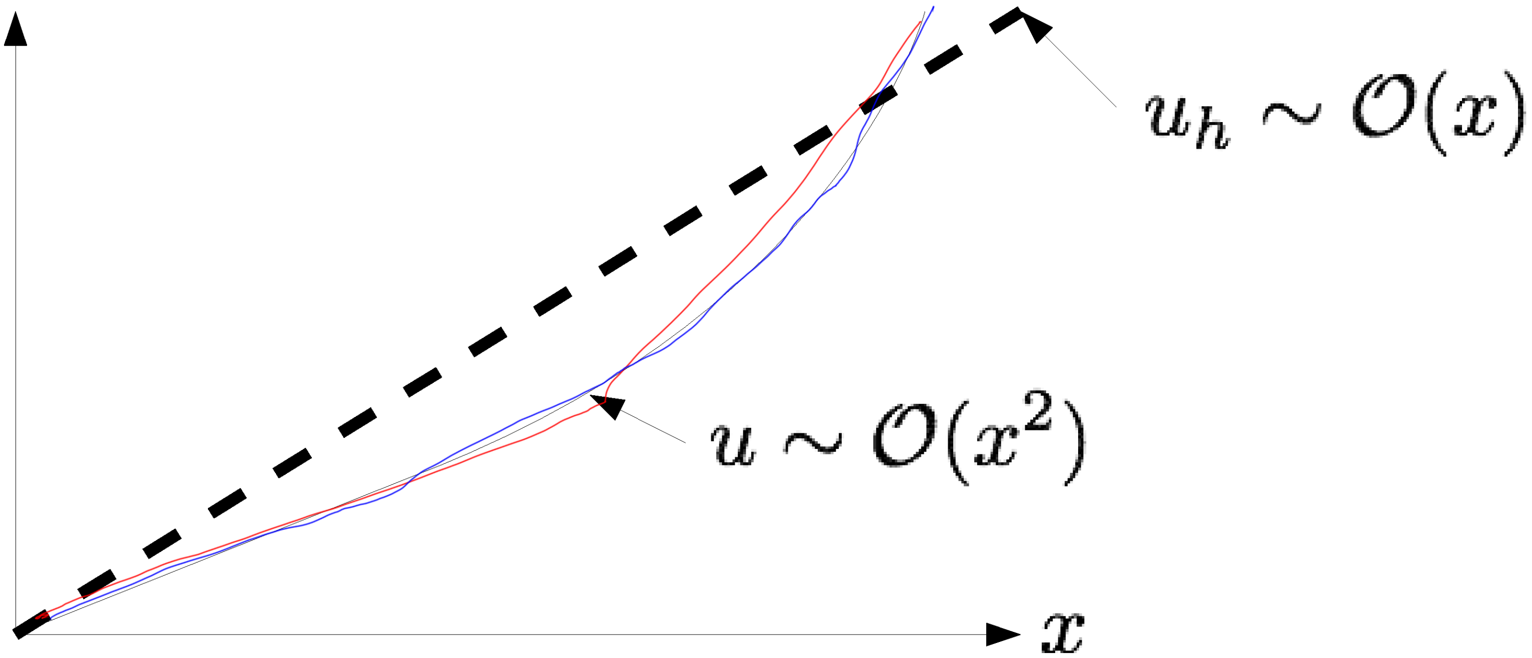
Sources of error

- Error due to the approximation of the domain – *discretization error*



Sources of error

- Error due to the approximation of the domain – *discretization error*
- Error due to approximation of the solution – truncation error



Sources of error

- Error due to the approximation of the domain – *discretization error*
- Error due to approximation of the solution – truncation error
- Computer related errors – roundoff error

Other remarks on FEM

- After assembly the resulting equations are usually singular, we have to impose boundary conditions in order to solve.
- For time-dependent problems there are two stages:
  - Use FEM to reduce PDE's to ODE's in time.
  - The ODE's in time are solved exactly or further approximated, typically with finite difference methods, to obtain algebraic equations which are then solved for the nodal values.

$$\underset{\sim}{K}\vec{u} = \vec{b}$$

$$\vec{u} = K^{-1}\vec{b}$$

In practice

- Analysis is done with FEA programs

SIMULIA

(ABAQUS)

ANSYS®

LSTC
Livermore Software
Technology Corp.

(LS-DYNA)

MSC Software®
Simulating Reality, Delivering Certainty™

(Nastran)

In practice

- Combined with solid modeling packages

•Catia

•NX (Unigraphics)

•Pro/Engineer

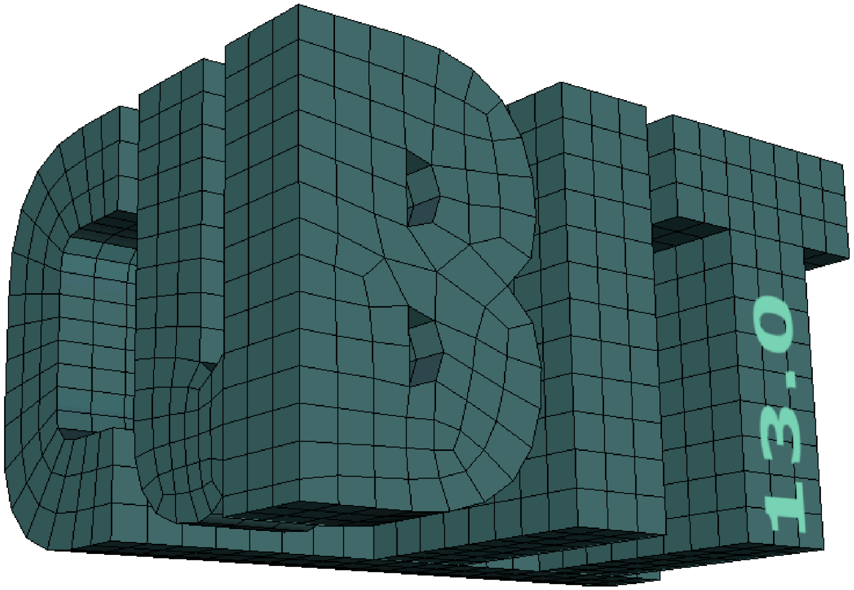
•Autodesk Inventor

In practice

- Combined with meshing packages

HyperWorks® 11

(Hypermesh)



Cool links

http://www.youtube.com/watch?v=geUCvKayhHE

http://www.youtube.com/watch?v=HmIcUc3A_5Y&feature=related