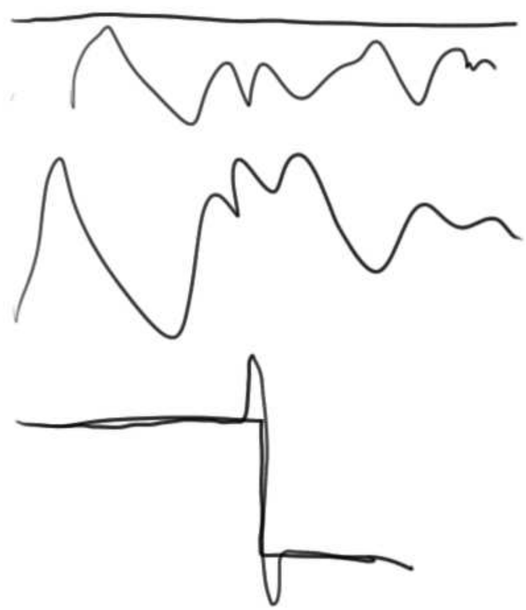
$$\frac{d}{dx}\left[EA \frac{du}{dx} \right] = 0 \qquad A(x) = A_0\left(1 - \frac{x}{2L}\right)$$

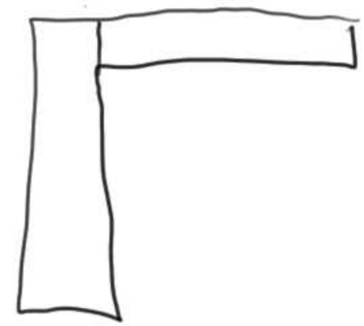
$$u(0) = 0 \qquad EA \left.\frac{\partial u}{\partial x}\right|_{x=L} = P$$

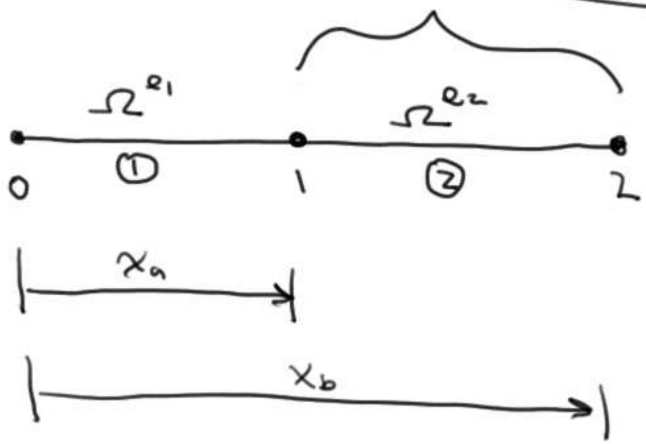
$$I(u) = \int_0^L \frac{EA}{2}\left(\frac{du}{dx}\right)^2 dx - Pu$$

$$u \approx u^h = \sum_i c_i x^i$$

$E(x) =$







$$u^h = c_1 + c_2 X$$

$$\text{Node 1}: \; u(x_a) = c_1 + c_2 x_a \equiv u_1^{e_2}$$

$$\text{Node 2}: \; u(x_b) = c_1 + c_2 x_b \equiv u_2^{e_2}$$

$$\underbrace{\begin{Bmatrix} u_1^{e_2} \\ u_2^{e_2} \end{Bmatrix}}_{\vec{u}^{e_2}} = \underbrace{\begin{bmatrix} 1 & x_a \\ 1 & x_b \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}}_{\vec{c}}$$

$$\vec{c} = A^{-1} \vec{u}$$

$$\vec{c} = \begin{bmatrix} \dfrac{u_2^{e_2} x_a - u_1^{e_2} x_b}{x_a - x_b} \\ \dfrac{u_1^{e_2} - u_2^{e}}{x_a - x_b} \end{bmatrix} \begin{matrix} \rightarrow c_1 \\ \\ \rightarrow c_2 \end{matrix}$$

$$u = c_1 + c_2 X$$

$$u^n = \frac{X_2^{e_2} x_a - u_1^{e_2} x_b}{x_a - x_b} + \frac{u_1^{e_2} - u_2^{e_2}}{x_a - x_b} X$$

$$u^n = \sum_j u_j N_j = u_1^{e_2} N_1 + u_2^{e_2} N_2$$

$$= u_1^{e_2} \underbrace{\left[\frac{x - x_b}{x_a - x_b}\right]}_{N_1} + u_2^{e_2} \underbrace{\left[\frac{x_a - x}{x_a - x_b}\right]}_{N_2}$$

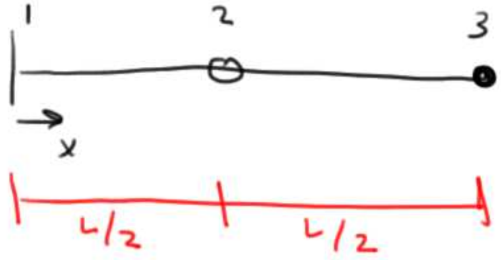
Let $x_b - x_a = L$

$$= u_1^{e_2}\left[1 - \frac{x}{L}\right] + u_2^{e_2}\left[\frac{x}{L}\right]$$

$$= \underbrace{\left[1 - \frac{x}{L} \quad , \quad \frac{x}{L}\right]}_{\vec{N}^T \to \text{shape function matrix}} \begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix}$$

Quadratic interpolant

Ω^{e^2}



Let $\vec{X}^T = [1 \quad x \quad x^2]$

$$u^h = N_j u_j = c_1 + c_2 x + c_3 x^2 = \vec{X}^T \vec{c}$$

$$\underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{\vec{u}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & L/2 & (L/2)^2 \\ 1 & L & L^2 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}}_{\vec{c}}$$

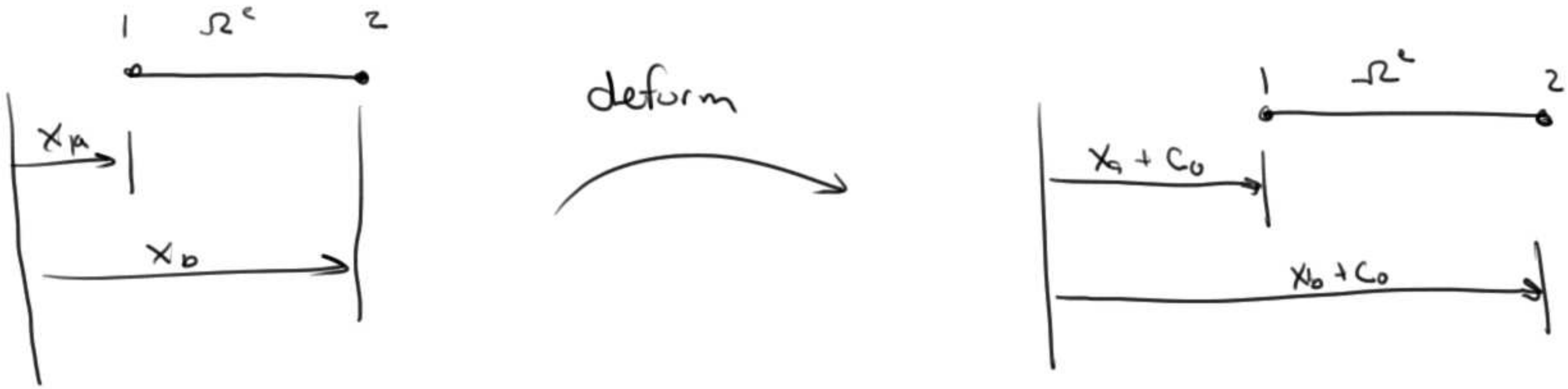
$$N^T \vec{u} = \vec{X}^T \vec{c}$$

$$N^T A \cancel{\vec{c}} = \vec{X}^T \cancel{\vec{c}}$$

$$N^T \overset{I}{A \cancel{A^{-1}}} = X^T A^{-1}$$

$$N^T = X^T A^{-1} = X^T \begin{bmatrix} X^T\big|_{x=x_1} \\ X^T\big|_{x=x_2} \\ X^T\big|_{x=x_3} \end{bmatrix}^{-1}$$

$$-a(x) \frac{\partial^2 u}{\partial x} + c(x) u = f(x)$$



$u_1 = c_0$ & $u_2 = c_0$ $\qquad u^h = c_0$

$$u^h = \frac{c_0}{c_0} = N_1 u_1 + N_2 u = \frac{N_1 c_0 + N_2 c_0}{c_0}$$

$$1 = N_1 + N_2$$ Partition-of-Unity

$$N_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i = j \end{cases} = \delta_{ij} \rightarrow$$ Kronecker Delta Property