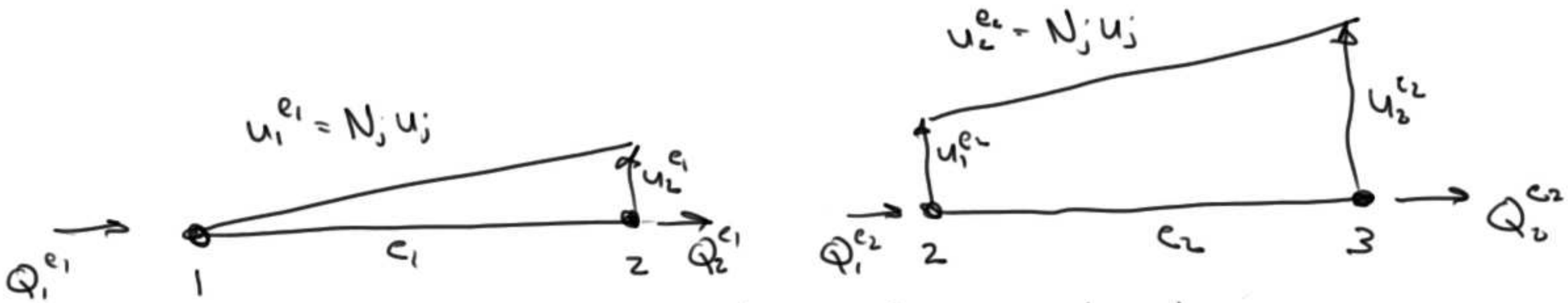
2n unknowns

$$(u_1, u_2, u_3, \ldots, u_n) + (Q_1, Q_2, \ldots, Q_n)$$

Continuity in u_i's + "balance" Q_i's



$$Q_2^{e_1} + Q_1^{e_2} = \begin{cases} 0 & \text{if no external source} \\ Q_J & \text{if external source of mag. } Q_J \end{cases}$$

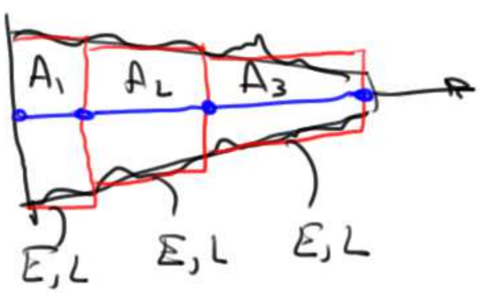
$$u_2^{e_1} = u_1^{e_2} = u_2$$

$$K^{e_1} = \begin{bmatrix} K_{11}^1 & K_{12}^1 \\ K_{21}^1 & K_{22}^1 \end{bmatrix} \quad K^{e_2} = \begin{bmatrix} K_{11}^2 & K_{12}^2 \\ K_{21}^2 & K_{22}^2 \end{bmatrix} \quad \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix} = K^G$$

$$[K^e]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_1^1 \\ f_2^1 + f_1^2 \\ f_2^2 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ Q_2^1 + Q_1^2 \\ Q_2^2 \end{Bmatrix}$$

$$f = \int_{x_a}^{x_b} f(x) N_j \, dx$$

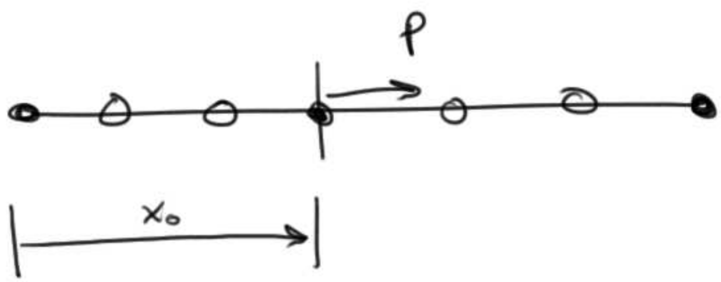
Ex



$$\begin{array}{cccc} 1 & 0 & & \\ \end{array}$$

$$\begin{bmatrix} \cancel{\frac{A_1E}{L}} & -\cancel{\frac{A_1E}{L}} & 0 & 0 \\ -\frac{A_1E}{L} & \frac{(A_1+A_2)E}{L} & -\frac{A_2E}{L} & 0 \\ 0 & -\frac{A_2E}{L} & \frac{(A_2+A_3)E}{L} & -\frac{A_3E}{L} \\ 0 & 0 & -\frac{A_3E}{L} & \frac{A_3E}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ P \end{Bmatrix}$$

What happens if Q_J is not at a node?



"function"

$$f(x) = P\Delta(x-x_0)$$

$$\int_{-\infty}^{\infty} \Delta(x-x_0)F(x)\,dx = F(x_0)$$

$$f_i^e = \int_{x_1}^{x_n} f(x)N_i(x)\,dx = \int_{x_1}^{x_n} P\Delta(x-x_0)N_i(x)\,dx = PN_i(x_0)$$

$$N_1 = \left(1-\frac{x}{L}\right) \qquad N_2 = \frac{x}{L}$$

$$f_1 = P\left(1-\frac{x_0}{2}\right), \quad f_2 = P\left(\frac{x_0}{L}\right) \qquad x_0 = \frac{L}{2}$$

$$= \frac{P}{2} \qquad\qquad = \frac{P}{2}$$

$$P_1 = A\sigma_1 = AE\varepsilon_1 = AE\left[\frac{u_1 - u_2}{L}\right] = \frac{AE}{L}u_1 - \frac{AE}{L}u_2$$

$$P_2 = A\sigma_2 = AE\varepsilon_2 = AE\left[\frac{u_2 - u_1}{L}\right] = -\frac{AE}{L}u_1 + \frac{AE}{L}u_2$$

$$\begin{bmatrix} \frac{AE}{L} & -\frac{AE}{L} \\ -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

$$-\frac{d}{dx}\left(a\frac{du}{dx}\right)+cu-f=0 \qquad 0<x<L$$

$$u(0)=u_0 \quad \& \quad \left(a\frac{\partial u}{\partial x}\right)_{x=L}=Q_0$$

	u	a	c	f	Q
Heat Transfer	Temp., T	Thermal Cond. K	conv	heat gen.	Heat, Q
Flow	Press, P	Resistance $\frac{K}{\mu}$	0	dist gen.	Point, P
Elasticity	Disp, u	stiffness AE	0	Axial force	Point Load

Ex

$a = 1 \quad , \quad c = 1 \quad , \quad f = -x^2$

$$-\frac{d^2u}{dx^2} - u + x^2 = 0 \qquad 0 < x < 1$$

$$u(0) = 0 \qquad u(1) = 0$$

$$K_{ij} = \int_{x_a}^{x_b} \left(\frac{dN_i}{dx}\frac{dN_j}{dx} - N_i N_j \right) dx$$

$$f = \int_{x_a}^{x_b} \left(-x^2\right) N_i \, dx$$