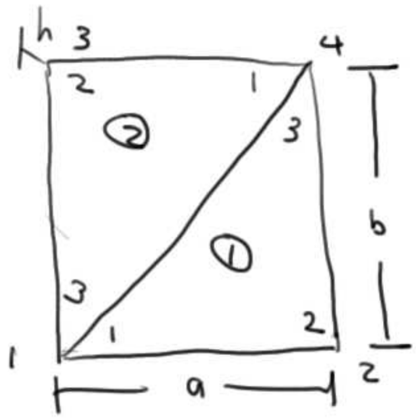


$$\text{connect} = \begin{bmatrix} 1, & 2, & 4 \\ 4, & 3, & 1 \end{bmatrix}$$

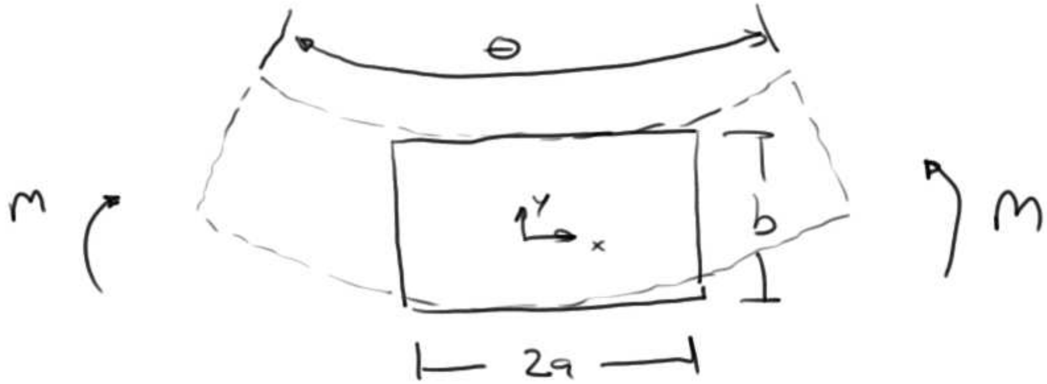
dof

$1 \to 1, 2$

$2 \to 3, 4$

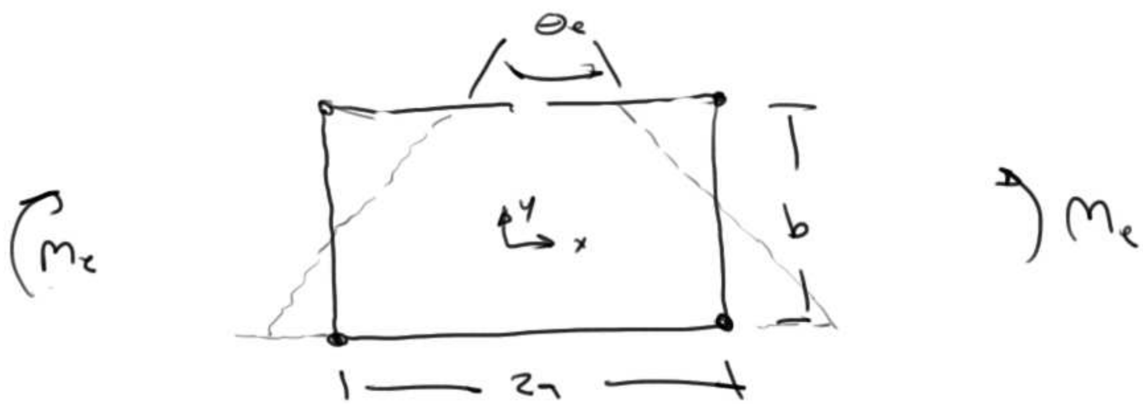
$3 \to 5, 6$

$4 \to 7, 8$



Euler beam

$$\varepsilon_{xx} = \frac{-\theta y}{2a}, \quad \varepsilon_{yy} = \nu \frac{\theta y}{2a}, \quad \varepsilon_{xy} = 0$$



$$\varepsilon_{xx} = -\frac{\theta_e y}{2a}, \quad \varepsilon_{yy} = 0, \quad \varepsilon_{xy} = -\frac{\theta_e x}{2a}$$ parasitis shear

Elastic strain energy

$$U = \frac{1}{2}\int [\varepsilon]^T C [\varepsilon]\, dV \quad \text{where } [\varepsilon] = [\, \varepsilon_{xy} \;\; \varepsilon_{yy} \;\; \varepsilon_{xy} \,]^T$$

$$U = \frac{M\theta}{2} \qquad U_e = \frac{M_e \theta_e}{2}$$

$M = M_e$ under plane stress $[C] \rightarrow$ plane stress

$$\frac{\theta_e}{\theta} = \frac{1-\nu^2}{1+\frac{1-\nu}{2}\left(\frac{a}{b}\right)^2}$$

where the term $\left(\frac{a}{b}\right)^2$ is present due to parasitic shear

$$\lim_{\frac{a}{b}\to\infty} \frac{1-\nu^2}{1+\frac{1-\nu}{2}\left(\frac{a}{b}\right)^2} = 0 \qquad \frac{\theta_e}{\theta} \approx 0 \quad \text{as } \frac{a}{b} \rightarrow \infty$$

the mesh "locks"

Consider volumetric strain

$$\frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy}$$

Under plain strain conditions, the pressure

$$p = \frac{E_y \, \theta \,(y - \nu)}{2a(1+\nu)(2\nu-1)}$$

$\nu \to \frac{1}{2}$ $\quad p \to \infty$ $\quad$ volumetric locking

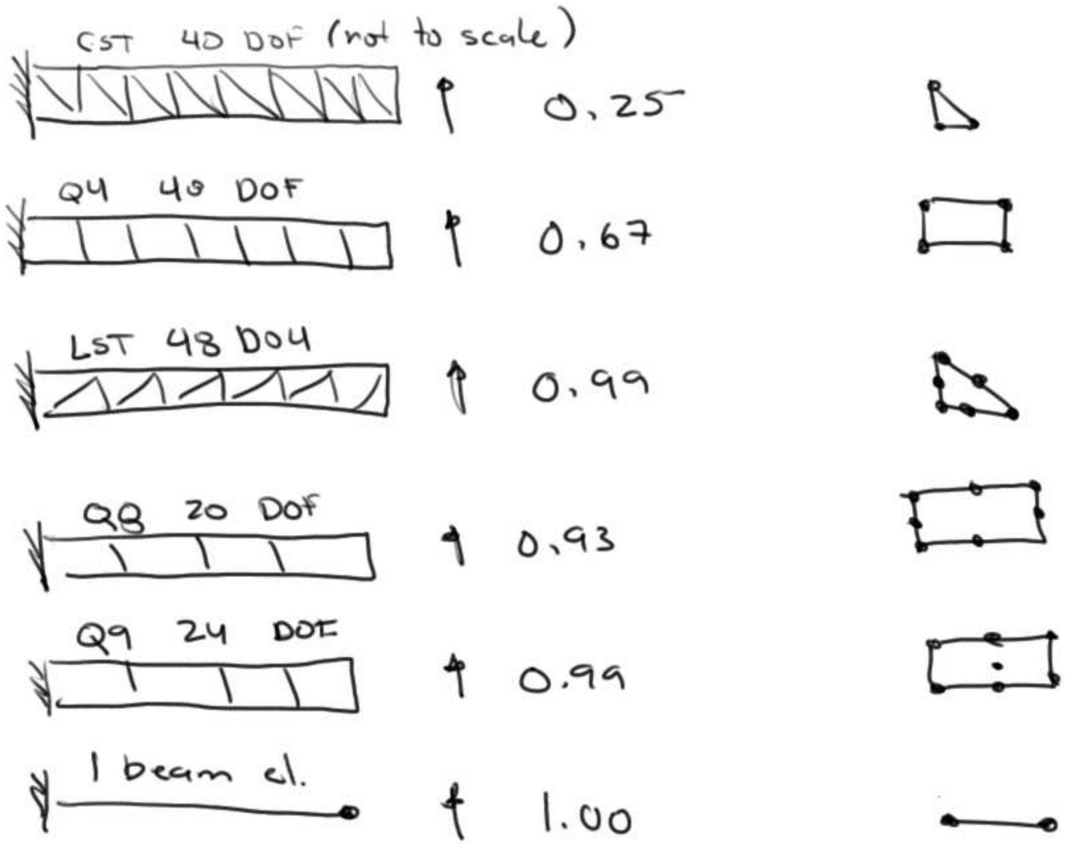
$$p_c = \frac{E_y \; y \; \theta_c}{2a(-1+\nu+2\nu^2)}$$

$p_e \to \infty$, as $\nu \to \frac{1}{2}$

"locking" $\neq$ immovability

$=$ excessive stiffness

For example (Q8)



$$1$$

$$\xi \quad \eta \quad \text{— One point}$$

$$\xi^2 \quad \xi\eta \quad \eta^2$$

$$\xi^3 \quad \xi^2\eta \quad \xi\eta^2 \quad \eta^3 \quad \text{— } 2 \times 2 \text{ point}$$

$$\xi^4 \quad \xi^3\eta \quad \xi^2\eta^2 \quad \xi\eta^3 \quad \eta^4$$

$$\xi^3\eta^2 \quad \xi^2\eta^3$$

$$\xi^3\eta^3$$

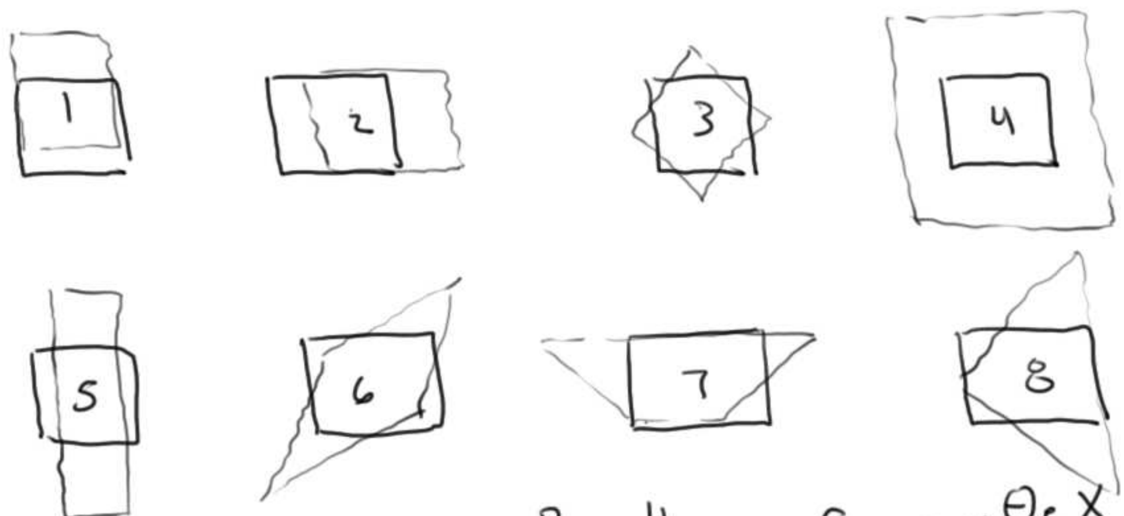
Define "full integration" = quadrature rule of sufficient accuracy to exactly integrate all coefficients of k^e of the undistorted element

$$k = \int_{\Omega^e} B^T [C] B J_3 h_e \, d\Omega \Rightarrow O(\xi^2 \eta^2) \quad \text{for Q4} \qquad 2 \times 2 \text{ rule}$$

Under integration

AKA "reduced integration"

- Use a rule that is less than exact
- Reduces computation time
- Can offset parasitic shear
- Introduce a defect i.e. spurious modes, singular modes, zero-energy modes hourglass modes.



Recall: $\varepsilon_{xy} = -\frac{\theta_e X}{2a}$ in bending