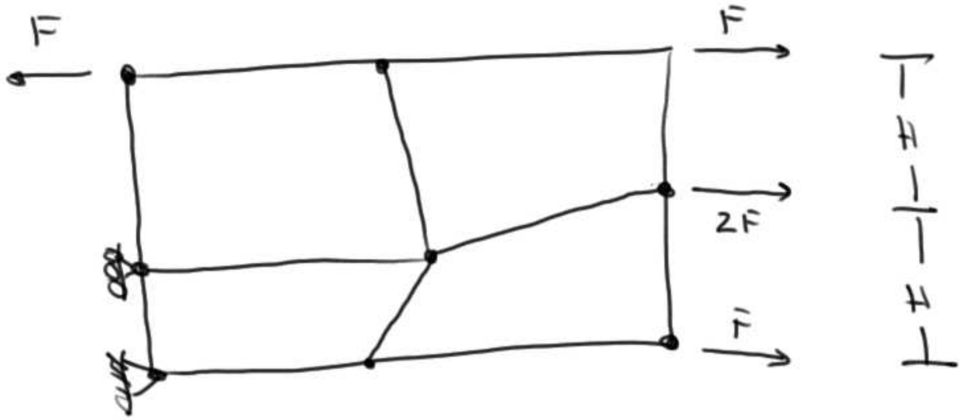
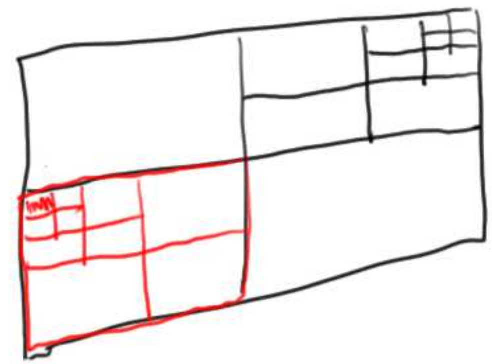
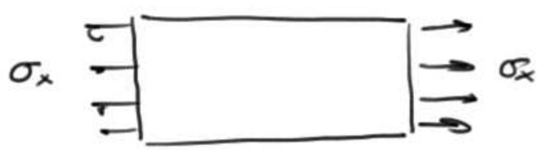
Patch test

Weak patch test







$$F = \frac{1}{2}(\sigma_x H t)$$

"Passes the patch test" $\Longrightarrow$ converge to exact solution

$$0 = \left[\int_\Omega h_e\, B^T C B\, d\vec{x}\right]\vec{u} + \left[\int_\Omega h_e \rho [N]^T [N]\, d\vec{x}\right]\ddot{\vec{u}} - \int h_e \rho [N]^T \vec{b}\, d\vec{x}$$

$$- \oint_\Gamma h_e [N]^T \vec{t}\, dS$$

$$\vec{\sigma} = C B \vec{u} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \qquad \sigma = \begin{bmatrix} \sigma_{xy} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix}$$

$$\sigma_{ij} = \sigma_{ij}' - \alpha \delta_{ij} p \qquad \text{where } \vec{m} = [1\ 1\ 1\ 0\ 0\ 0]^T \text{ in 3D}$$

$$\vec{\sigma} = \vec{\sigma}' - \alpha \vec{m} p \qquad \vec{m} = [1\ 1\ 0]^T \text{ in 2D}$$

$$p \approx p^h = N_j^p p_j$$

$$0 = \Big[\underbrace{\int_\Omega h_e B^T C B \, d\vec{x}}_{K}\Big]\vec{u} - \Big[\underbrace{\int_\Omega B^T \alpha \vec{m} N^p \, d\vec{x}}_{Q}\Big]\vec{p} + \Big[\underbrace{\int_\Omega \rho [N^u]^T [N^u] \, d\vec{x}}_{\hat{M}}\Big]\ddot{\vec{u}}$$

$$\underbrace{- \int h_e \rho [N^u]^T \vec{b} \, d\vec{x} - \oint h_e [N^u]^T \vec{t}}_{F^{(1)}} \, dS$$

$$0 = \Big[\underbrace{\int_\Omega h_e B^T \alpha \vec{m} N^p \, d\vec{x}}_{Q}\Big]\dot{\vec{u}} + \Big[\underbrace{\int_\Omega h_e (\nabla N^p)^T \frac{\bar{\bar{K}}}{\mu} \nabla N^p \, d\vec{x}}_{H}\Big]\vec{p} + \Big[\underbrace{\int_\Omega h_e N^p \frac{1}{M} N^p \, d\vec{x}}_{S}\Big]\dot{\vec{p}}$$

$$\underbrace{+ \int h_e (\nabla N^p)^T \nabla^T \left(\frac{\bar{\bar{K}}}{\mu} \rho \vec{b}\right) d\vec{x} - \oint (N^p)^T \vec{q}}_{F^{(2)}} \, dS$$

where $\frac{1}{M} = \frac{\Phi}{K_f} + \frac{\alpha - \Phi}{K_s}$

$$\begin{bmatrix} \tilde{M} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\vec{u}} \\ \ddot{\vec{p}} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ Q^T & S \end{bmatrix} \begin{Bmatrix} \dot{\vec{u}} \\ \dot{\vec{p}} \end{Bmatrix} + \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \begin{Bmatrix} \vec{u} \\ \vec{p} \end{Bmatrix} - \begin{Bmatrix} F^{(1)} \\ F^{(2)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

drained behavior "sand & gravel" high-permeability

$$\begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \begin{Bmatrix} \vec{u} \\ \vec{p} \end{Bmatrix} = \begin{Bmatrix} F^{(1)} \\ F^{(2)} \end{Bmatrix}$$

undrained behavior "silts & clays" low-permeability

$$H = 0 \qquad F^{(2)} = 0$$

$$p = \rho^2 \frac{\partial e}{\partial \rho}$$

$$Q^T \dot{\vec{u}} + S \vec{p} = 0 \quad \Rightarrow \quad \vec{u}(t=0) = \vec{p}(t=0) = \vec{0}$$

$$Q^T \vec{u} + S \vec{p} = 0$$

$$\begin{bmatrix} K & -Q \\ Q^T & \end{bmatrix} \begin{Bmatrix} \vec{u} \\ \vec{p} \end{Bmatrix} = \begin{Bmatrix} \vec{F}^{(1)} \\ 0 \end{Bmatrix}$$