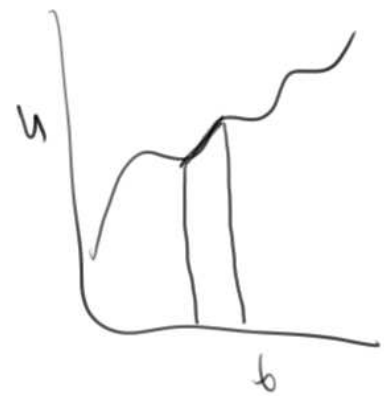
Implicit

Newmark - β



$$u^{n+1} = \tilde{u}^{n+1} + \beta \Delta t^2 a^{n+1}$$

$$\tilde{u}^{n+1} = u^n + \Delta t v^n + \frac{\Delta t^2}{2}(1 - 2\beta) a^n$$

$$v^{n+1} = \tilde{v}^{n+1} + \gamma \Delta t a^{n+1}$$

$$\tilde{v}^{n+1} = v^n + (1-\gamma)\Delta t a^n$$

β

$\gamma \to$ damping parameter

Solve $a^{n+1} = \frac{1}{\beta \Delta t^2}(u^{n+1} - \tilde{u}^{n+1})$ for $\beta > 0$

$\beta = 0$, $\gamma = \frac{1}{2}$ explicit central difference scheme

$\beta = \frac{1}{2}$, $\gamma = \frac{1}{2}$ undamped trapezoid rule

$\gamma > \frac{1}{2}$ damped response $\propto (\gamma - \frac{1}{2})$

Unconditionally stable

$\beta \geq \frac{\gamma}{2} \geq \frac{1}{4}$

$$0 = R = \frac{1}{\beta \Delta t^2} M(u^{n+1} - \tilde{u}^{n+1}) - f^{ext} - f^{int}$$

$$K^T_{ij} = \frac{\partial R_i}{\partial u_j} = \frac{1}{\beta \Delta t^2} M_{ij} + \frac{\partial f^{ext}_i}{\partial u_j} - \frac{\partial f^{int}_i}{\partial u_j}$$

$$f^{int}_i = K_{ij} u_j \qquad \cancel{\frac{\partial K_{ij}}{\partial u_k}} u_j + K_{ij} \frac{\partial u_j}{\partial u_k} = K_{ij} \delta_{jk} = K_{ik}$$

$$K = \int B^T C B \, h_e \, dA$$

$$f^{int} = K(u) = \int B^T \sigma \, h_e \, dA$$

Flowchart for implicit

$F = f^{int} + f^{ext}$

1. Initialize v^0, u^0, σ^0, $n=0$, $t=0$
2. Compute F^0
3. Compute $a^n = M^{-1} F^0$
4. Estimate $u_{NEW} = u^n$ or $u_{NEW} = \tilde{u}^{n+1}$
5. Newton Interate
   a) Compute (u_{NEW})
   b) $a^{n+1} = \frac{1}{\beta \Delta t^2}(u_{NEW} - \tilde{u}^{n+1})$
   c) $R = M a^{n+1} - F^0$
   d) $K^T = \frac{\partial R}{\partial \vec{u}}$
   e) Modify K^T for B.C's
   f) Solve $\Delta u = -(K^T)^{-1} R$
   g) Check convergence
6. Update disp $u^{n+1} = u_{NEW}$
7. Check Energy Balace

$$[M]\ddot{u}_{n+1} + \left[\int B^T \sigma' \, d\Omega\right] - [Q]p_{n+1} - f^{(1)}_{n+1} = 0$$

$$[Q]\dot{\vec{u}}_{n+1} + [H]p_{n+1} + [S]\dot{\vec{p}}_{n+1} - f^{(1)}_{n+1} = 0$$

Extend β - Method

$$\ddot{\vec{u}}_{n+1} = \ddot{\vec{u}}_n + \Delta\ddot{\vec{u}}_{n+1/2}$$

$$\dot{\vec{u}}_{n+1} = \dot{\vec{u}}_n + \ddot{\vec{u}}_n \Delta t + \beta_1 \Delta\ddot{\vec{u}}_n \Delta t$$

$$\vec{u}_{n+1} = \vec{u}_n + \dot{\vec{u}}_n \Delta t + \frac{1}{2}\ddot{\vec{u}}_n \Delta t^2 + \frac{1}{2}\beta_2 \Delta\ddot{\vec{u}}_n \Delta t^2$$

$$\dot{\vec{p}}_{n+1} = \dot{\vec{p}}_n + \Delta\dot{\vec{p}}_{n+1/2}$$

$$\vec{p}_{n+1} = \vec{p}_n + \dot{\vec{p}}_n \Delta t + \gamma \Delta\dot{\vec{p}}_n \Delta t$$

Uncoditionally stablu

$$\beta_2 \geq \beta_1 \geq \gamma \geq \frac{1}{2}$$

$$R^{(1)} = M_{n+1}\,\Delta\ddot{u}_{n+1/2} + P(\vec{u}_{n+1}) - Q_{n+1}\,\gamma\,\Delta t\,\Delta\dot{p}_{n+1/2} - F^{(1)}_{n+1}$$

$$R^{(2)} = Q^{T}_{n+1}\,\beta_1\,\Delta t\,\Delta\ddot{u}_{n+1/2} + H_{n+1}\,\gamma\,\Delta t\,\Delta\dot{p}_{n} + S_{n+1}\,\Delta\dot{p}_{n+1/2} - F^{(2)}_{n+1}$$

$$K^{T} = \begin{bmatrix} \dfrac{\partial R^{(1)}}{\partial \Delta\ddot{u}_n} & \dfrac{\partial R^{(1)}}{\partial \Delta\dot{p}} \\[2ex] \dfrac{\partial R^{(2)}}{\partial \Delta\ddot{u}_n} & \dfrac{\partial R^{(2)}}{\partial \Delta\dot{p}} \end{bmatrix}$$

Use Newton method to solve for $\Delta\ddot{u}_{n+1/2}$, $\Delta\dot{p}_{n+1/2}$