$$K(\vec{U})\,\vec{U} = \vec{F}$$

Use iteration:

$$K(U^n)\,U^{n+1} = F$$

$$U^{n+1} = [K(U^n)]^{-1} F$$

$$\frac{|U^{n+1} - U^n|}{|U^{n+1}|} < \varepsilon \quad \text{say } 10^{-6}$$

Newton - Raphson

$$\vec{R} \equiv K(\vec{U})\,\vec{U} - F = 0$$

$$\vec{R} = \vec{R}^n + \left(\frac{\partial \vec{R}}{\partial \vec{U}}\right)_n \left(\vec{U}^{n+1} - \vec{U}^n\right) + \frac{1}{2!}\left(\frac{\partial^2 \vec{R}}{\partial \vec{U}^2}\right)_n \left(\vec{U}^{n+1} - \vec{U}^n\right)^2 + \ldots$$

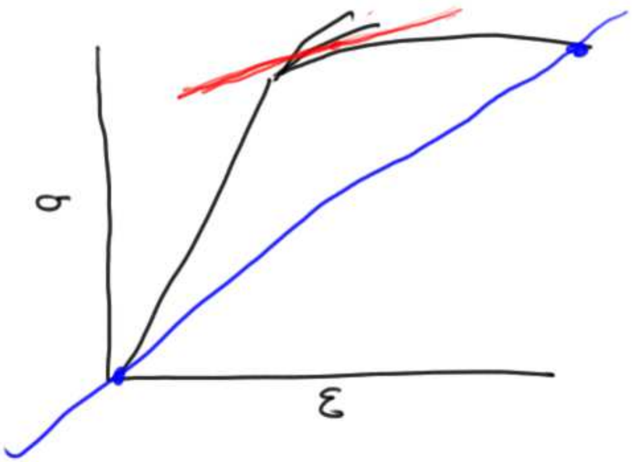
$$0 \approx R^n + K_T^n \Delta \vec{U} + O(\Delta \vec{U}^2)$$

K_T is the tangent stiffness matrix

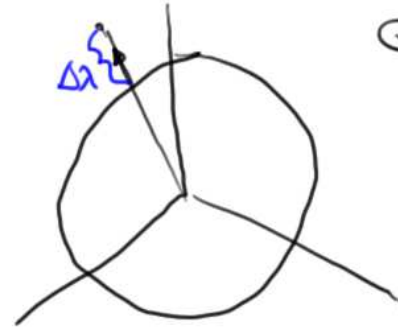
$K_T = \dfrac{\partial \vec{R}}{\partial \vec{U}}$ evaluated at $U = U^n$

$$\Delta \vec{U} = -(K_T)^{-1} R^n = (K_T(U^n))^{-1}\left(F - K(U^n)U^n\right)$$

$$\vec{U}^{n+1} = \vec{U}^n + \Delta \vec{U}$$



$K(U)U = F$



$$Q_{ij} = \frac{S_{ij}}{\sqrt{S_{ij} S_{ij}}}$$

$$S = \sqrt{S_{ij} S_{ij}}$$

From 9/23/2015 - 9/25/2015

$$\dot{\varepsilon}^d_{ij} Q_{ij} + \frac{\dot{S}}{2\mu} - \dot{\lambda} = 0$$

$$\frac{\Delta\varepsilon^d_{ij}}{\cancel{\Delta t}} Q_{ij} + \frac{\Delta S}{\cancel{\Delta t}} \frac{1}{2\mu} - \frac{\Delta\lambda}{\cancel{\Delta t}} = 0 \Rightarrow \Delta\varepsilon^d_{ij} Q_{ij} + \frac{S_{n+1} - S_n}{2\mu} - \Delta\lambda = 0$$

$S_n \rightarrow$ known

$S_{n+1} \rightarrow \sqrt{\frac{2}{3}}\, Y_{n+1}$

$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \therefore \quad S = \begin{bmatrix} \frac{2}{3}Y & & \\ & -\frac{1}{3}Y & \\ & & -\frac{1}{3}Y \end{bmatrix}$$

$$Y = Y(\varepsilon^p, \dot{\varepsilon}^p) = Y(\dot{\varepsilon}^p, \sqrt{\tfrac{2}{3}}\ \dot{\lambda})$$

$$\frac{\Delta \varepsilon^p}{\cancel{\Delta t}}$$

$$Y_{n+1} = Y\left(\varepsilon^p_n, \sqrt{\tfrac{2}{3}}\ \frac{\Delta\lambda}{\Delta t}\right)$$

$$\varepsilon^p_{n+1} = \varepsilon^p_n + \dot{\varepsilon}^p \cancel{\Delta t}$$

$$= Y\left(\varepsilon^p_{n+1}, \sqrt{\tfrac{2}{3}}\ \frac{\Delta\lambda}{\Delta t}\right)$$

$$= Y\left(\varepsilon^p_n + \sqrt{\tfrac{2}{3}}\,\Delta\lambda, \sqrt{\tfrac{2}{3}}\ \frac{\Delta\lambda}{\Delta t}\right)$$

$$\Delta\varepsilon^d_{ij} Q_{ij} - \Delta\lambda - \frac{1}{2\mu}\left[\sqrt{\tfrac{2}{3}}\, Y\left(\varepsilon^p_n + \sqrt{\tfrac{2}{3}}\,\Delta\lambda, \sqrt{\tfrac{2}{3}}\ \frac{\Delta\lambda}{\Delta t}\right) - S_n\right] = 0$$

$$Y = \sigma_y\left(1 + \frac{\varepsilon^p}{\varepsilon^p_0}\right)^n \left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}^p}\right)^m$$

$$\dot{\sigma} = C\,\dot{\varepsilon}$$

$$\sigma_{n+1} - \sigma_n = C(\varepsilon_{n+1} - \varepsilon_n)$$

$$S_{n+1} = \begin{cases} S^{tr}_{ij} & f^{tr} < 0 \\ \sqrt{\tfrac{2}{3}}\, Y_{n+1} Q^{tr} & f^{n+1} = 0 \end{cases}$$

$$\sigma_{n+1} = S_{n+1} + p_{n+1} I, \quad p_{n+1} = p_n + K\,\dot{\varepsilon}_{kk}\,\Delta t$$

$$K_T = \frac{\partial \vec{R}}{\partial \vec{U}} = \frac{\partial R(\vec{U} + h\hat{e}_i) - \partial R(\vec{U})}{h}$$