$$\{\sigma\} = [C]\{\varepsilon\}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_y}{\partial y} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{Bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}}^{\text{Dmat}} \underbrace{\begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} \\ \frac{\partial u_y}{\partial y} \end{Bmatrix}}_{A}$$

$$\frac{\partial u_x}{\partial x} = \frac{\partial N_I}{\partial x} u_{xI}$$

$$\xi(x,y) \leftrightarrow x(\xi,\eta)$$
$$\eta(x,y) \leftrightarrow y(\xi,\eta)$$

$$x = N_I x_I$$

$$N(\xi,\eta)$$

$$N(x(\xi,\eta), y(\xi,\eta))$$

$$\frac{\partial N}{\partial \xi} = \frac{\partial N}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial N}{\partial y}\frac{\partial y}{\partial \xi}$$
$$\frac{\partial N}{\partial \eta} = \frac{\partial N}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial N}{\partial y}\frac{\partial y}{\partial \eta}$$

$$\Rightarrow \begin{Bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{Bmatrix} = \overbrace{\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}}^{J} \begin{Bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{Bmatrix} \Rightarrow \begin{Bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_x}{\partial y} \end{Bmatrix} = \begin{bmatrix} & & \\ & J & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial u_x}{\partial \xi} \\ \frac{\partial u_x}{\partial \eta} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{\partial u_y}{\partial x} \\ \frac{\partial u_y}{\partial y} \end{Bmatrix} = \begin{bmatrix} & & \\ & J & \\ & & \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial u_y}{\partial \xi} \\ \frac{\partial u_y}{\partial \eta} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} \\ \frac{\partial u_y}{\partial y} \end{Bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} & J & \end{bmatrix}^{-1} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{bmatrix} & J & \end{bmatrix}^{-1} \end{bmatrix}}_{\text{Jmat}} \begin{Bmatrix} \frac{\partial u_x}{\partial \xi} \\ \frac{\partial u_x}{\partial \eta} \\ \frac{\partial u_y}{\partial \xi} \\ \frac{\partial u_y}{\partial \eta} \end{Bmatrix}$$

$$\begin{Bmatrix} \frac{\partial u_x}{\partial \xi} \\ \frac{\partial u_x}{\partial \eta} \\ \frac{\partial u_y}{\partial \xi} \\ \frac{\partial u_y}{\partial \eta} \end{Bmatrix} = \overbrace{\begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \cdots \\ \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \cdots \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & \cdots \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & \cdots \end{bmatrix}}^{\text{Nmat}} \begin{Bmatrix} u_x^1 \\ u_y^1 \\ u_x^2 \\ u_y^2 \\ u_x^3 \\ u_y^3 \\ \vdots \\ u_x^n \\ u_y^n \end{Bmatrix}$$

$$\vec{\varepsilon} = \underbrace{\text{Dmat} \cdot \text{Jmat} \cdot \text{Nmat}}_{B} \cdot \vec{u}$$

$$\vec{\varepsilon} = B\,\vec{u}$$