$$R^{(1)} \equiv [M]\ddot{\vec{u}} + \left(\int B^T \sigma' \, d\Omega\right) - [Q]\vec{p} - \int (N^u)^T \rho \vec{b} \, d\Omega + \int (N^u)^T \vec{t} \, d\Gamma = 0$$

$$R^{(2)} \equiv [Q]\dot{\vec{u}} + [H]\vec{p} + [S]\dot{\vec{p}} + \int (N^p)^T \nabla \cdot \left(\frac{\bar{\bar{K}}}{\mu} \rho_f \vec{b}\right) + \int (N^p)^T \vec{q} \, d\Gamma = 0$$

$$K^T = \begin{bmatrix} \frac{\partial R^{(1)}}{\partial \Delta \ddot{\vec{u}}} & \frac{\partial R^{(1)}}{\partial \Delta \dot{\vec{p}}} \\ \frac{\partial R^{(2)}}{\partial \Delta \ddot{\vec{u}}} & \frac{\partial R^{(2)}}{\partial \Delta \dot{\vec{p}}} \end{bmatrix} \qquad K^T \vec{x} = b \qquad \vec{x} = \begin{Bmatrix} \Delta \ddot{\vec{u}} \\ \Delta \dot{\vec{p}} \end{Bmatrix}$$

$$\tilde{\vec{R}} = \vec{R}^{(1)} + \vec{R}^{(2)} \qquad \vec{x} = \begin{Bmatrix} u_x^1 \\ u_y^1 \\ p^1 \\ \vdots \\ u_x^n \\ u_y^n \\ p^n \end{Bmatrix}$$

$$K^T = \frac{\partial R}{\partial \vec{x}} \approx \frac{\partial R_i(\vec{x} + \delta \hat{e}_j) - R(\vec{x})}{\delta}$$

$$r = \|R\|_{L_2}$$

Jacobian - Free Newton - Krylov

Benefit from preconditioning

$$\underbrace{K^{-1}K}\vec{x} = K^{-1}\vec{b}$$

$$I\vec{x} = K^{-1}\vec{b}$$

$$\vec{x} = K^{-1}\vec{b}$$

$$K^{-1} \approx M$$

$$K_{ii} = \frac{\partial R_i(\vec{x} + \delta\hat{e}_i) - R_i}{\delta}$$

$$\underbrace{MK}_{\approx I}\vec{x} = M\vec{b}$$

$$\begin{bmatrix} \times & \times & \times & \times & & & & \\ \times & \times & \times & \times & \times & & & \\ & \times & \times & \times & \times & \times & & \\ & & \times & \times & \times & \times & \times & \\ & & & \times & \times & \times & \times & \times \\ & & & & & & & \\ & & & & & & & \end{bmatrix}$$