

$$\phi c_t \frac{\partial \rho(\vec{x})}{\partial t} = \nabla \cdot \left(\frac{\bar{k}}{\mu} \nabla \rho(\vec{x}) \right)$$

$$\int_{\Omega} \omega(\vec{x}) \phi c_t \frac{\partial \rho(\vec{x})}{\partial t} = \nabla \cdot \left(\frac{\bar{k}}{\mu} \nabla \rho(\vec{x}) \right) d\vec{x}$$

$$\int_{\Omega} \int_{\Omega} \omega(x,y) \phi(x,y) c_t \frac{\partial \rho(x,y)}{\partial t} - \omega(x,y) \nabla \cdot \left(\frac{\bar{k}}{\mu} \nabla \rho(x,y) \right) dz dA = 0$$

$$\int_{\Omega} \int_{\Omega} \omega(x,y) d(x,y) \phi c_t \frac{\partial \rho(x,y)}{\partial t} - \omega(x,y) d(x,y) \nabla \cdot \left(\frac{\bar{k}}{\mu} \nabla \rho(x,y) \right) dA = 0$$

$$\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \left\{ \int_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} \int_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2} \omega(x,y) \phi c_t \frac{\partial \rho(x,y)}{\partial t} - \omega(x,y) d(x,y) \left[\frac{\partial}{\partial x} \left(\frac{\bar{k}}{\mu} \nabla \rho(x,y) \right) + \frac{\partial}{\partial y} \left(\frac{\bar{k}}{\mu} \nabla \rho \right) \right] dx dy \right\} = 0$$

$$\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \left\{ \int_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} \int_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2} w(x,y) \phi_c d(x,y) \frac{\partial p(x,y)}{\partial t} dx dy \right.$$

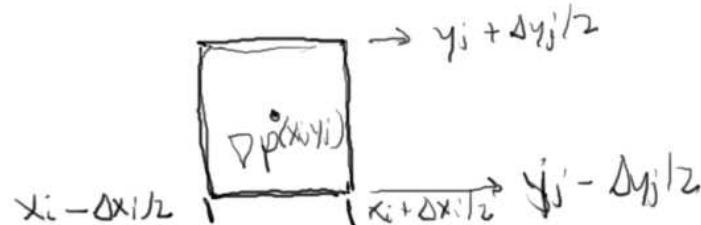
$$+ \int_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} \int_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2} \frac{\partial}{\partial x} (w(x,y) d(x,y)) \cdot \frac{k}{\mu} \nabla p dx dy$$

$$+ \int_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} \int_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2} \frac{\partial}{\partial y} (w(x,y) d(x,y)) \cdot \frac{k}{\mu} \nabla p dx dy$$

$$- \left[\int_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2} w(x,y) d(x,y) \frac{k}{\mu} \nabla p dy \right]_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} - \left[\int_{x_i - \Delta x_i/2}^{x_i + \Delta x_i/2} w(x,y) \frac{k}{\mu} \nabla p dx \right]_{y_j - \Delta y_j/2}^{y_j + \Delta y_j/2}$$

Let $w(x,y) = 1$ + use midpoint quadrature on integrals

Use f_p for ∇p



$$\nabla p = \left\{ \begin{array}{l} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \end{array} \right\}$$

$$\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \left\{ \frac{\phi c + d_{i,j}}{B_s \mu} \frac{\partial p_{i,j}}{\partial t} \Delta x_i \Delta y_j - d_{i+\frac{1}{2},j} y_i \Delta y_j \frac{k_{i+\frac{1}{2},j}}{B_s \mu} \left(\frac{p_{i+1,j} - p_{i,j}}{\Delta x_{i+\frac{1}{2}}} \right) \right.$$

$$+ d_{i+\frac{1}{2},j} y_i \Delta y_j \frac{k_{i-\frac{1}{2},j}}{B_s \mu} \left(\frac{p_i - p_{i-1}}{\Delta x_{i+\frac{1}{2}}} \right) - d_{i,j+\frac{1}{2}} \Delta x_i \frac{k_{i,j+\frac{1}{2}}}{B_s \mu} \left(\frac{p_{i,j+1} - p_{i,j}}{\Delta y_{j+\frac{1}{2}}} \right)$$

$$+ d_{i,j-\frac{1}{2}} \Delta x_i \frac{k_{i,j-\frac{1}{2}}}{B_s \mu} \left(\frac{p_{i,j} - p_{i,j-1}}{\Delta y_{j-\frac{1}{2}}} \right) \} = 0$$

$$B_{i,j} = \frac{\phi_{i,j} c + V_{i,j}}{B\alpha} \quad \text{where } V_{i,j} = d_{i,j} \Delta x_i \Delta y_j$$

$$T_{i+\frac{1}{2},j} = \frac{d_{i+\frac{1}{2},j} \Delta y_j}{\Delta x_{i+\frac{1}{2}}} \quad \frac{k_{i+\frac{1}{2},j}}{B\alpha M}$$

$$T_{i-\frac{1}{2},j} = \frac{d_{i-\frac{1}{2},j} \Delta y_j}{\Delta x_{i-\frac{1}{2}}} \quad \frac{k_{i-\frac{1}{2},j}}{B\alpha M}$$

$$T_{i,j+\frac{1}{2}} = \frac{d_{i,j+\frac{1}{2}} \Delta x_i}{\Delta y_{i+\frac{1}{2}}} \quad \frac{k_{i,j+\frac{1}{2}}}{B\alpha \mu}$$

$$T_{i,j-\frac{1}{2}} = \frac{d_{i,j-\frac{1}{2}} \Delta x_i}{\Delta y_{i-\frac{1}{2}}} \quad \frac{k_{i,j-\frac{1}{2}}}{B\alpha \mu}$$

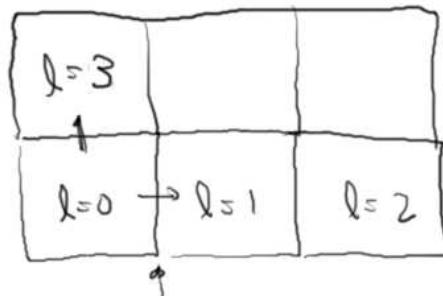
$$\sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} \left\{ B_{i,j} \frac{\partial p_{i,j}}{\partial t} + T_{i+\frac{1}{2},j} (p_{i,j} - p_{i+1,j}) + T_{i-\frac{1}{2},j} (p_{i,j} - p_{i-1,j}) + T_{i,j+\frac{1}{2}} (p_{i,j} - p_{i,j+1}) + T_{i,j-\frac{1}{2}} (p_{i,j} - p_{i,j-1}) \right\} = 0$$

$$B_{0,0} \frac{\partial p_{0,0}}{\partial t} + B_{1,0} \frac{\partial p_{1,0}}{\partial t} + B_{2,0} \frac{\partial p_{2,0}}{\partial t} + \dots + B_{N_x-1,0} \frac{\partial p_{N_x-1,0}}{\partial t} + B_{0,1} \frac{\partial p_{0,1}}{\partial t} + B_{1,1} \frac{\partial p_{1,1}}{\partial t} \\ + B_{N_x-1,N_y-1} \frac{\partial p_{N_x-1,N_y-1}}{\partial t} + T_{v_2,0} (P_{0,0} - P_{1,0}) + \dots$$

Recall $\ell = j \cdot N_x + l$

$$B_{0,0} \rightarrow B_{\ell=0} \quad \text{likewise} \quad P_{0,0} \rightarrow P_{\ell=0}$$

$$B_{1,0} \rightarrow B_{\ell=1} \quad " \quad P_{1,0} \rightarrow P_{\ell=1}$$



$N_x = 3$

$$B_{\ell=0} \frac{\partial p_{\ell=0}}{\partial t} + B_{\ell=1} \frac{\partial p_{\ell=1}}{\partial t} + \dots + T_{v_2,0} (P_{\ell=0} - P_{\ell=1}) + \dots + T_{0,1/2} (P_{\ell=0} - P_{\ell=N_x}) + \dots$$

$$\left[\begin{matrix} B_{0,0} \\ B_{0,1} \\ \vdots \\ B_{0,N_y} \end{matrix} \right] \left\{ \frac{\partial \vec{p}}{\partial t} \right\} + [T] \left\{ \vec{p} \right\} = \left\{ \vec{Q} \right\}$$

$B_{0,\ell} \in N_x \times N_y$

$$\begin{bmatrix} T_{-1/2,0} + T_{1/2,0} + T_{0,1/2} + T_{0,-1/2} \\ -T_{1/2,0} \\ \vdots \\ T_{1/2,0} + T_{3/2,0} + T_{0,-1/2} + T_{0,1/2} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}_{Nx}$$

$[T]^s$

$\left([T] + \frac{1}{\Delta t} [B] \right) \vec{p}^{n+1} = \left(\frac{1}{\Delta t} [B] \{ \vec{p}^n \} + \right)$

Exactly as in 10

Discrete in time "implicit" or "explicit"

<img alt="A diagram showing a large bracket labeled [T]^s containing several smaller brackets. One bracket is labeled Nx and contains a vertical stack of terms. Another bracket contains terms like T_{1/2,0}, T_{3/2,0}, etc. A third bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A fourth bracket contains terms like T_{-1/2,0}, T_{1/2,0}, etc. A fifth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A sixth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A seventh bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A eighth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A ninth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A tenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A eleventh bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twelfth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A thirteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A fourteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A fifteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A sixteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A seventeenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A eighteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A nineteenth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twentieth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-first bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-second bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-third bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-fourth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-fifth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-sixth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-seventh bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-eighth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A twenty-ninth bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A thirty bracket contains terms like T_{0,1/2}, T_{0,-1/2}, etc. A thirty-one bracket contains 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