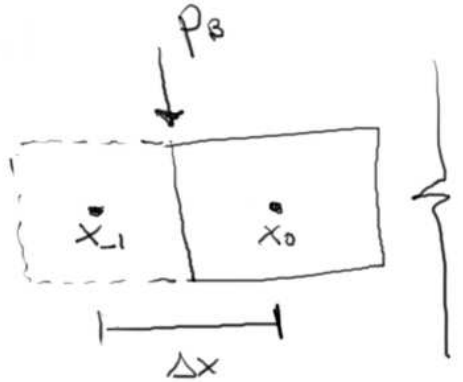
Prescribed Pressure (Direchlet Boundary Condition)



$$p_B = \frac{p_{-1} + p_0}{2} \qquad \eta = \frac{\alpha \Delta t}{\Delta x^2}$$

$$\rightarrow p_{-1} = 2p_B - p_0$$

$$\frac{\alpha \Delta t}{\Delta x^2}\left(-p_{-1} + 2p_0 - p_1\right)[I]\Delta p + \eta \begin{bmatrix} 3 & -1 & 0 & 0 & \cdots \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{Bmatrix} p_0 \\ p_1 \\ \vdots \\ p_{N-1} \end{Bmatrix} = \{\vec{\delta}\}$$

$$\eta\left(-2p_B + p_0 + 2p_0 - p_1\right)$$

$$\eta\left(3p_0 - 2p_B - p_1\right)$$

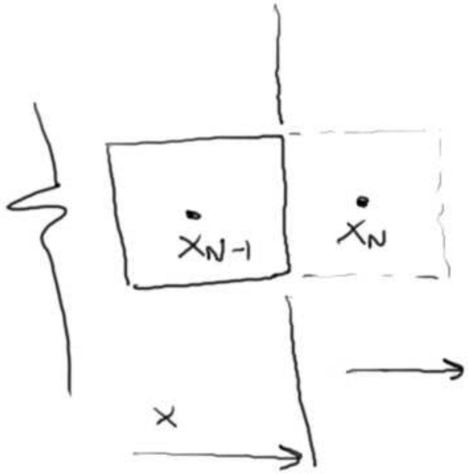
$$[I]\{\Delta\vec{P}\} + \eta[A]\{\vec{P}\} = \{\vec{P}_B\}$$

$$A = \begin{bmatrix} 3 & -1 & 0 & 0 & \cdots \\ -1 & 2 & -1 & 0 & \cdots \\ 0 & -1 & 2 & -1 & \cdots \\ \vdots & & \ddots & \ddots & 3 \end{bmatrix}$$

$$\vec{P}_B = \begin{Bmatrix} 2\eta P_{BL} \\ 0 \\ 0 \\ \vdots \\ 2\eta P_{BR} \end{Bmatrix}$$

Prescribed pressure on right boundary

Prescribed flux



$\rightarrow q$ if $q = 0$ No flow

$$q = -\frac{k}{\mu}\frac{\partial P}{\partial x} = -\frac{k}{\mu}\frac{(P_N - P_{N-1})}{\Delta x} = \frac{k}{\mu}\frac{(P_{N-1} - P_N)}{\Delta x}$$

$$\boxed{p_N = p_{N-1} - \frac{q\mu\Delta x}{k}}$$

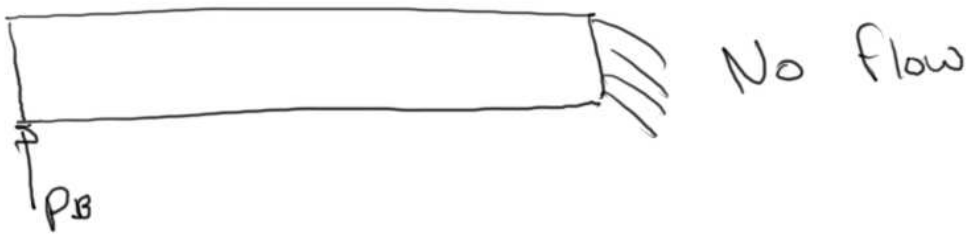
$$\boxed{p_N = p_{N-1}} \text{ No flow}$$

$$A = \begin{bmatrix} & & \\ & & \\ & & 1 \end{bmatrix}$$

$\eta(-p_{N-2} + 2p_{N-1} - p_N)$

$\eta\left(-p_{N-2} + p_{N-1} + \frac{q\mu\Delta x}{k}\right)$

$$\vec{p}_B = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \frac{q\mu\eta\Delta x}{k} \end{Bmatrix}$$



$$[I]\{\Delta\vec{P}\} + \eta[A]\{\vec{P}\} = \{\vec{P}_B\}$$

Prescribe Pressure

Where

$$A = \begin{bmatrix} 3 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & \\ & -1 & 2 & -1 & \cdots & \\ & & \ddots & \ddots & & \\ & & & -1 & 2 & -1 \\ 0 & & & & -1 & 1 \end{bmatrix}$$

$$+ \{\vec{P}_B\} = \begin{Bmatrix} 2\eta P_B \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

No flow

$$[I]\{\vec{p}^{\,n+1}\} - [I]\{\vec{p}^{\,n}\} + \eta[A]\{\vec{p}^{\,n}_{n+1}\} = \{\vec{p}_B\}$$

$$\{\vec{p}^{\,n+1}\} = \left([I] - \eta[A]\right)\{\vec{p}^{\,n}\} + \{\vec{p}_B\}$$ Explicit

$$\{\vec{p}^{\,n+1}\} = \left([I] + \eta[A]\right)^{-1}\left(\{\vec{p}^{\,n}\} + \{\vec{p}_B\}\right)$$ Implicit