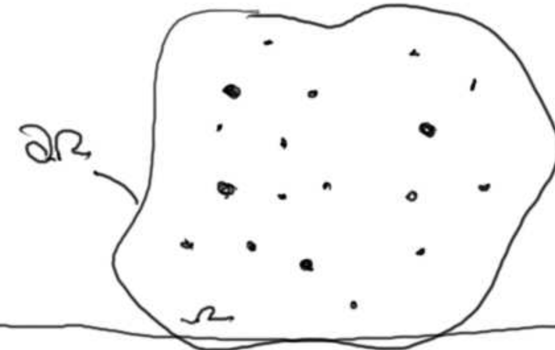
$$\frac{\partial p}{\partial t} = \alpha \; \nabla \cdot (\nabla p)$$

$$\int_\Omega w(\vec{x}) \frac{\partial p(\vec{x},t)}{\partial t} - w(\vec{x})\alpha \; \nabla \cdot \left(\nabla p(\vec{x},t)\right) d\vec{x} = 0$$

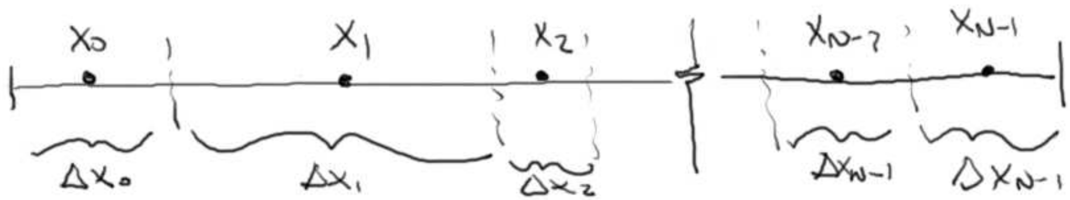


$$\int_\Omega w(\vec{x}) \frac{\partial p(\vec{x},t)}{\partial t} d\vec{x} + \alpha \int_\Omega \nabla w(\vec{x}) \nabla p(\vec{x},t) \, d\vec{x} + \alpha \int_{\partial\Omega} w(\vec{x}) \nabla p(\vec{x},t) \, d\vec{x} = 0$$

Weak Form

↓ 1D

$$\int_0^L w(x) \frac{\partial p(x,t)}{\partial t} dx + \alpha \int_0^L \frac{\partial w(x)}{\partial x} \frac{\partial p(x,t)}{\partial x} dx + \alpha \left[w(x) \frac{\partial p(x,t)}{\partial x} \right]_0^L$$



$$\sum_{i=0}^{N-1}\left[\int_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2} \omega(x)\frac{\partial p(x,t)}{\partial t}\,dx + \alpha\int_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\frac{\partial \omega}{\partial x}\frac{\partial p}{\partial x}\,dx + \alpha\left[\omega(x)\frac{\partial p}{\partial x}\right]_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\right]$$

$$\omega(x) = \delta(x - x_i)$$

Delta Function (Dirac Delta Function)

$$\delta(x=0) = \infty$$
$$\delta(x\neq 0) = 0$$

$$\int_{-\infty}^{\infty}\delta(x)\,dx = 1$$

$$\int_{a-\varepsilon}^{a+\varepsilon} f(x)\,\delta(x-a)\,dx = f(a)$$

$$\int_{a-\varepsilon}^{a+\varepsilon} f(x)\frac{\partial}{\partial x}\delta(x-a)\,dx = -\frac{\partial f(a)}{\partial x}$$

$$\sum_{i=0}^{N-1}\left[\frac{\partial p(x_i,t)}{\partial t} - \alpha\frac{\partial^2 p(x_i,t)}{\partial x^2}\right] = 0$$