Finite Difference Approxi's

$f(x)$

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \frac{1}{3!}\frac{\partial^3 f}{\partial x^2}\Delta x^3 + \dots$$

$$p(x+\Delta x) = p(x) + \boxed{\frac{\partial p}{\partial x}}\Delta x + \frac{1}{2!}\frac{\partial^2 p}{\partial x^2}\Delta x^2 + \dots$$

$$\frac{\partial p}{\partial x} = \frac{p(x+\Delta x) - p(x)}{\Delta x} - \underbrace{\frac{1}{2!}\frac{\partial^2 p}{\partial x^2}\Delta x + \dots}_{O(\Delta x)}$$

Forward difference

$$p(x-\Delta x) = p(x) - \boxed{\frac{\partial p}{\partial x}}\Delta x + \frac{1}{2!}\frac{\partial^2 p}{\partial x^2}\Delta x^2 - \dots$$

→ Backward difference

$$\frac{\partial p}{\partial x} = \frac{p(x) - p(x-\Delta x)}{\Delta x} + \underbrace{\frac{1}{2!}\frac{\partial^2 p}{\partial x^2}\Delta x}_{O(\Delta x)} - \dots$$

$O(\Delta x)$ ← First-order accurate

$$p(x+\Delta x) = \cancel{p(x)} + \cancel{\frac{\partial p}{\partial x}\Delta x} + \cancel{\frac{1}{2!}\frac{\partial^2 p}{\partial x^2}}\Delta x^2 + \cancel{\frac{1}{3!}\frac{\partial^3 p}{\partial x^3}}\Delta x^3 + \dots$$

$$-+p(x-\Delta x) = \cancel{p(x)} - \cancel{\frac{\partial p}{\partial x}\Delta x} + \cancel{\frac{1}{2!}\frac{\partial^2 p}{\partial x^2}}\Delta x^2 - \cancel{\frac{1}{3!}\frac{\partial^3 p}{\partial x^3}}\Delta x^3 + \dots$$

$$p(x+\Delta x) - p(x-\Delta x) = 2\boxed{\frac{\partial p}{\partial x}}\Delta x + 2\,\frac{1}{3!}\frac{\partial^3 p}{\partial x^3}\Delta x^3 + \dots$$

$$\boxed{\frac{\partial p}{\partial x} = \frac{p(x+\Delta x) - p(x-\Delta x)}{2\Delta x}} - \underbrace{\frac{1}{3!}\frac{\partial^3 p}{\partial x^3}\Delta x^2}_{O(\Delta x^2)}$$

Central-diff. approximation

$O(\Delta x^2) \rightarrow$ second-order accurate

2nd-derivative

$$p(x+\Delta x) + p(x-\Delta x) = 2p(x) + 2\,\frac{1}{2!}\boxed{\frac{\partial^2 p}{\partial x^2}}\Delta x^2 + 2\,\frac{1}{4!}\frac{\partial^4 p}{\partial x^4}\Delta x^4$$

$$\boxed{\frac{\partial^2 p}{\partial x^2} = \frac{p(x+\Delta x) - 2p(x) + p(x-\Delta x)}{\Delta x^2}} - \underbrace{\frac{1}{12}\frac{\partial^4 p}{\partial x^4}\Delta x^2}_{O(\Delta x^2)}$$

Central-diff.

$O(\Delta x^2) \rightarrow$ second-order accurate