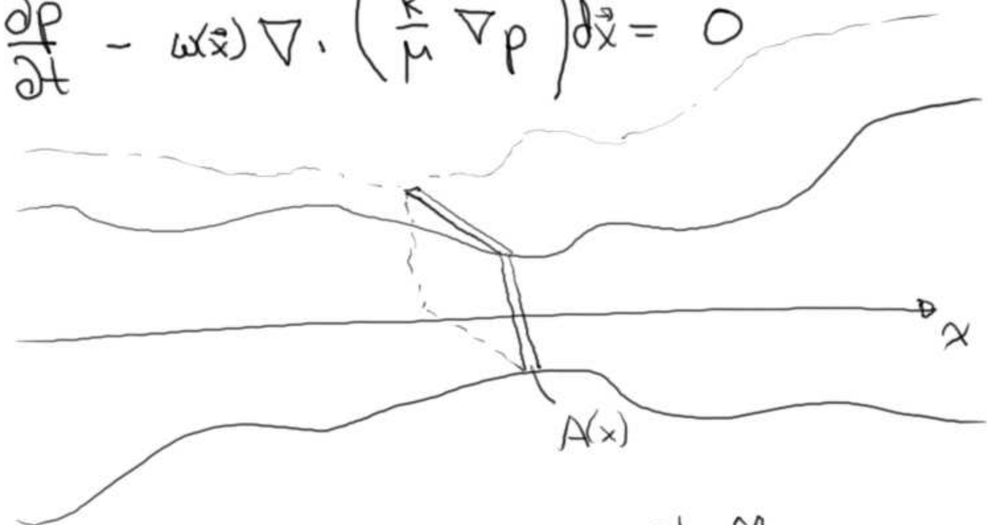
$$\phi c_t \frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{\bar{\bar{k}}}{\mu} \nabla p \right) \Rightarrow$$ For homo., iso. $$\frac{\partial p}{\partial t} = \alpha \nabla \cdot (\nabla p)$$

$$\int_\Omega \omega(\vec{x}) \phi c_t \frac{\partial p}{\partial t} - \omega(\vec{x}) \nabla \cdot \left(\frac{\bar{\bar{k}}}{\mu} \nabla p \right) d\vec{x} = 0$$



$$\int_0^L \iint_{A(x)} \omega(x) \phi c_t \frac{\partial p(x,t)}{\partial t} \, dA' \, dx - \int_0^L \iint_{A(x)} \omega(x) \nabla \cdot \left(\frac{\bar{\bar{k}}}{\mu} \nabla p(x,t) \right) dA' \, dx = 0$$

$$\int_0^L A(x) \omega(x) \phi c_t \frac{\partial p(x,t)}{\partial t} dx - \int_0^L A(x) \omega(x) \frac{\partial}{\partial x} \left(\frac{k(x)}{\mu(x)} \frac{\partial p(x,t)}{\partial x} \right) dx = 0$$

$$\int_0^L A(x) \omega(x) \phi c_t \frac{\partial p(x,t)}{\partial t} dx + \int_0^L \frac{\partial}{\partial x} \left(A(x) \omega(x) \right) \frac{k(x)}{\mu(x)} \frac{\partial p(x,t)}{\partial x} dx - \left[A(x) \omega(x) \frac{k}{\mu} \frac{\partial p}{\partial x} \right]_0^L = 0$$

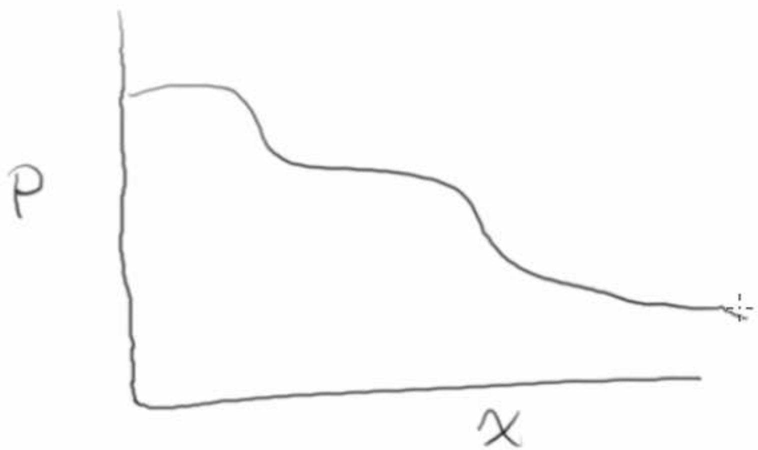
Let $w(x) = 1$

$$\int_0^L A\phi c_t \frac{\partial p}{\partial t}\,dx + \int_0^L \frac{k}{\mu}\frac{\partial A}{\partial x}\frac{\partial p}{\partial x}\,dx - \left[A\frac{k}{\mu}\frac{\partial p}{\partial x}\right]_0^L = 0$$

$$\sum_{i=0}^{N-1}\left\{\int_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2} A\phi c_t \frac{\partial p}{\partial t}\,dx + \int_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2} \frac{k}{\mu}\frac{\partial A}{\partial x}\frac{\partial p}{\partial x}\,dx - \left[A\frac{k}{\mu}\frac{\partial p}{\partial x}\right]_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\right\} = 0$$

Use midpoint quadrature $\int_a^b f(x)\,dx \approx (b-a)\,f\left(\frac{a+b}{2}\right)$

$$\sum_{i=0}^{N-1}\left\{A(x_i)\phi c_t \frac{\partial p(x_i,t)}{\partial t}\Delta x_i + \underbrace{\frac{k(x_i)}{\mu(x_i)}\frac{\partial A(x_i)}{\partial x}\frac{\partial p(x_i)}{\partial x}}\Delta x - \left[A\frac{k}{\mu}\frac{\partial p}{\partial x}\right]_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\right\} = 0$$

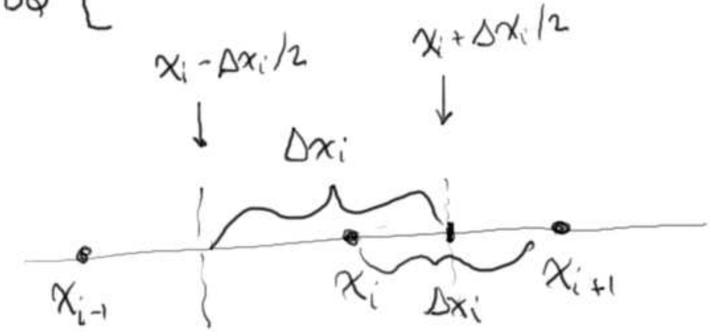


$$\sum_{i=0}^{N-1}\left\{\frac{V(x_i)\,c_t\phi}{B_\alpha}\,\frac{\partial p(x_i,t)}{\partial t} - \frac{1}{B_\alpha}\left[A(x)\,\frac{k(x)}{\mu(x)}\,\frac{\partial p(x)}{\partial x}\right]_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\right\} = 0$$

$$V(x_i) = A(x_i)\,\Delta x_i$$

$$\sum_{i=0}^{N-1}\left\{\frac{V(x_i)\,c_t\phi}{B_\alpha}\,\frac{\partial p(x_i,t)}{\partial t} - \frac{kA}{\mu B_\alpha}\left[\frac{\partial p(x)}{\partial x}\right]_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\right\} = 0$$

$$\sum_{i=0}^{N-1}\left\{\underbrace{\frac{V(x_i)\,c_t\phi}{B_\alpha}}_{B(x_i)}\,\frac{\partial p(x_i,t)}{\partial t} - \frac{kA}{\mu B_\alpha}\left[\frac{\partial p(x_i+\Delta x_i/2)}{\partial x} - \frac{\partial p(x_i-\Delta x_i/2)}{\partial x}\right]\right\} = 0$$



$$\sum_{i=0}^{N-1}\left\{B(x_i)\,\frac{\partial p(x_i,t)}{\partial t} - \overbrace{\frac{kA}{\mu B_\alpha \Delta x_i}}^{T(x_i)}\left[p(x_{i+1}) - p(x_i) - \big(p(x_i) - p(x_{i-1})\big)\right]\right\} = 0$$

$$p(x_{i+1}) - 2p(x_i) + p(x_{i-1})$$

$$\sum_{i=0}^{N-1}\left\{ B(x_i)\frac{\partial p(x_i,t)}{\partial t} + T(x_i)\left[-p(x_{i-1}) + 2p(x_i) - p(x_{i+1})\right]\right\} = 0$$

$$B(x_0)\frac{\partial p(x_0,t)}{\partial t} + T(x_0)\left[-p(x_{-1}) + 2p(x_0) - p(x_1)\right] +$$

$$B(x_1)\frac{\partial p(x_1,t)}{\partial t} + T(x_1)\left[-p(x_0) + 2p(x_1) - p(x_2)\right] + \dots +$$

$$B(x_{N-1})\frac{\partial p(x_{N-1},t)}{\partial t} + T(x_{N-1})\left[-p(x_{N-2}) + 2p(x_{N-1}) - p(x_N)\right] = 0$$

$$\underbrace{\begin{bmatrix} B_0 & & & & \\ & B_1 & & & \\ & & B_2 & & \\ & & & \ddots & \\ & & & & B_{N-1} \end{bmatrix}}_{[B]} \left\{\frac{\partial \vec{p}}{\partial t}\right\} + \underbrace{\begin{bmatrix} 3T_0 & -T_0 & 0 & \cdots & \\ -T_1 & 2T_1 & -T_1 & \cdots & \\ & -T_2 & 2T_2 & -T_2 & \\ & & & \ddots & \\ & & & & \ddots \end{bmatrix}}_{[T]} \{\vec{p}\} = \{\vec{Q}\}$$

$$\{\vec{Q}\} \rightarrow \begin{Bmatrix} 2T_0 p_B \\ 0 \\ 0 \\ \vdots \end{Bmatrix}$$

$$\boxed{p(x_{-1}) = 2p_B - p(x_0)}$$

$$\boxed{p_N = p_{N-1}}$$