$$\frac{D}{Dt}(\rho_0 \phi_0) = \frac{D}{Dt}(\rho \phi J)$$

$$0 = \frac{D}{Dt}(\rho \phi J) = \underbrace{\frac{D}{Dt}(\rho\phi)}\, J + \rho\phi \frac{D}{Dt}(J)$$

$$= J\left[\frac{\partial(\rho\phi)}{\partial t} + \nabla(\rho\phi)\cdot\vec{v}\right] + \rho\phi\frac{D}{Dt}(J)$$

$$= \cancel{J}\left[\frac{\partial(\rho\phi)}{\partial t} + \nabla(\rho\phi)\cdot\vec{v}\right] + \rho\phi\,\cancel{J}\,\nabla\cdot\vec{v}$$

$$= \frac{\partial(\rho\phi)}{\partial t} + \underbrace{\nabla(\rho\phi)\cdot\vec{v} + \rho\phi\,\nabla\cdot\vec{v}}_{\nabla\cdot(\rho\phi\vec{v})}$$

$$\boxed{0 = \frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot(\rho\phi\vec{v})}$$

$$\frac{D}{Dt}(J) = \frac{D}{Dt}(\det \bar{\bar{F}}) = \underbrace{\frac{\partial(\det \bar{\bar{F}})}{\partial F_{ij}}} \frac{\partial F_{ij}}{\partial t}$$

$$= \det \bar{\bar{F}} \, (F_{ji})^{-1} \frac{\partial F_{ij}}{\partial t}$$

$$= J \frac{\partial X_i}{\partial x_j} \frac{\partial}{\partial t}\left(\frac{\partial x_i}{\partial X_j}\right)$$

$$= J \frac{\partial X_i}{\partial x_j} \frac{\partial}{\partial X_j}\left(\underbrace{\frac{\partial x_i}{\partial t}}_{v_i}\right)$$

$$= J \frac{\partial X_i}{\partial x_j} \frac{\partial v_i}{\partial X_j}$$

$$= J \frac{\partial v_i}{\partial x_j} \frac{\partial X_i}{\partial X_j}$$

$$= J \frac{\partial v_i}{\partial x_j} \delta_{ij}$$

$$= J \frac{\partial v_i}{\partial x_i}$$

$$\boxed{\frac{D}{Dt}(J) = J \, \nabla \cdot \vec{v}}$$

$$\nabla = \left\{ \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3} \right\}$$

$$\vec{v} = \{ v_1, v_2, v_3 \}$$

$$\nabla \cdot \vec{v} = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3}$$

$$f = x$$

$$\frac{\partial f}{\partial x} = \frac{\partial x}{\partial x} = 1$$

Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial v_i}{\partial x_i} = \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i}$$

<u>Einstien</u> notation

$$0 = \frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\phi\rho v_1)}{\partial x_1} + \frac{\partial(\phi\rho v_2)}{\partial x_2} + \frac{\partial(\phi\rho v_3)}{\partial x_3}$$

$x_1 \to x$

$x_2 \to y$

$x_3 \to z$

$$\boxed{0 = \frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho\phi v_x)}{\partial x} + \frac{\partial(\phi\rho v_y)}{\partial y} + \frac{\partial(\rho\phi v_z)}{\partial z}}$$

Conservation of Mass