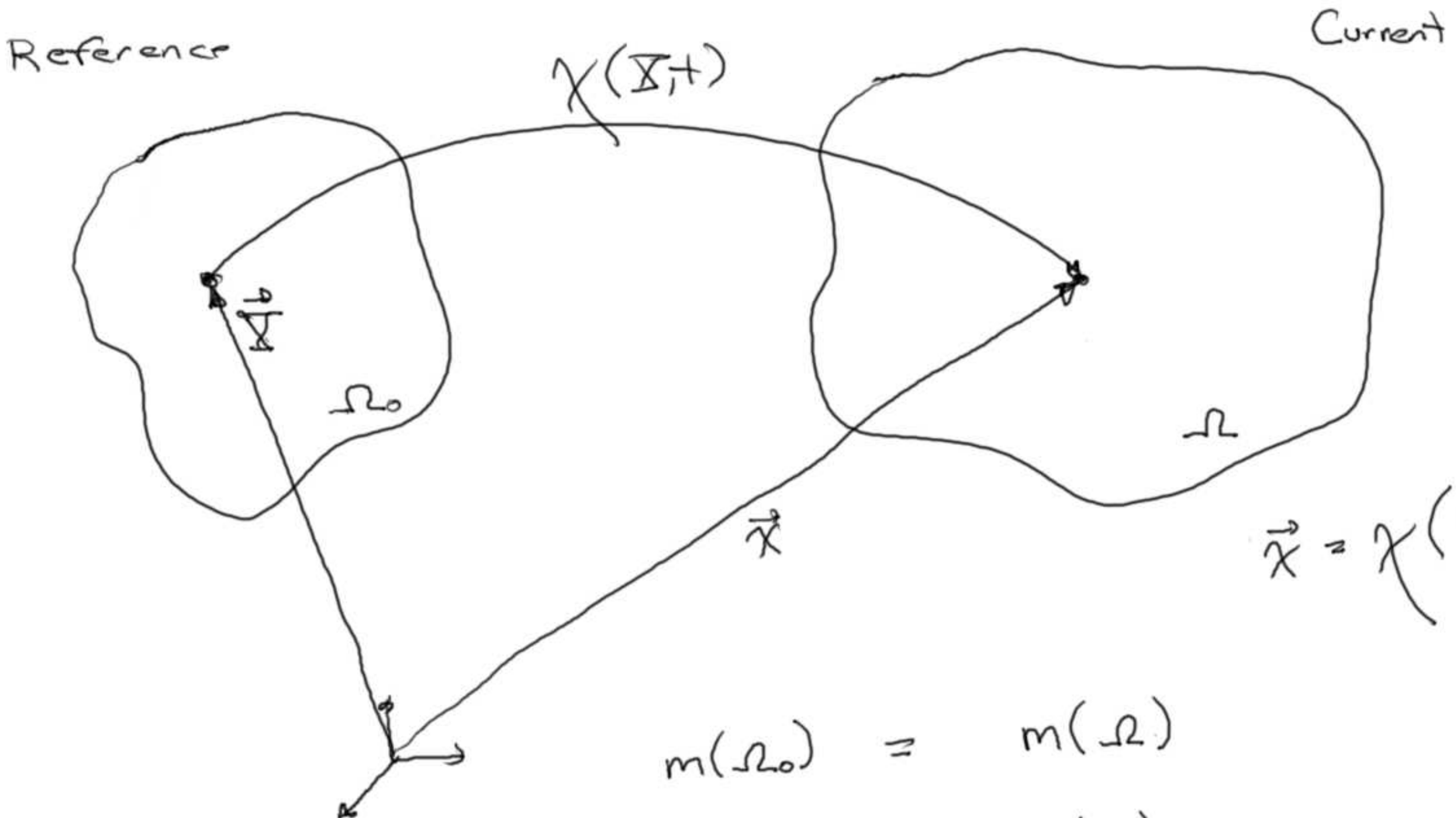

$$\vec{x} = \chi(\vec{X},t)$$

$$m(\Omega_0) = m(\Omega)$$

$$dm(\vec{X}) = dm(\vec{x})$$

$$\int_{\Omega_0} \rho_0(\vec{X})\, \phi_0(\vec{X})\, dV_0 = \int_{\Omega} \rho(\vec{x},t)\, \phi(\vec{x},t)\, dV$$

$$\iiint_{\Omega_0} \rho_0(\vec{X})\, \phi_0(\vec{X})\, dX_1\, dX_2\, dX_3 = \iiint_{\Omega} \rho(\vec{x},t)\, \phi(\vec{x},t)\, dx_1\, dx_2\, dx_3$$

$$\longrightarrow \iiint_{\Omega_0} \rho_0(\vec{X})\,\phi_0(X)\,dX_1\,dX_2\,dX_3 = \iiint_{\Omega_0} \rho(\vec{x},t)\,\phi(\vec{x},t)\,J\,dX_1\,dX_2\,dX_3$$

$$J = \det(\bar{\bar{F}}) = \begin{vmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{vmatrix} = \begin{vmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{vmatrix}$$

$$F_{ij} = \frac{\partial x_i}{\partial X_j}$$

$$\int_{\Omega_0} \rho_0(\vec{X})\,\phi_0(\vec{X})\,dV_0 = \int_{\Omega_0} \rho(\vec{x},t)\,\phi(\vec{x},t)\,J\,dV_0$$

$$\boxed{\rho_0(\vec{X})\,\phi_0(\vec{X}) = \rho(\vec{x},t)\,\phi(\vec{x},t)\,J}$$

Material for of conservation of mass