

Mixed Method

$$\theta [\text{Explicit}] + (1-\theta) [\text{Implicit}] \Rightarrow \text{Mixed Method}$$

$$0 < \theta < 1$$

$$\begin{aligned} \theta [T] \{ \vec{p}^n \} + (1-\theta) [T] \{ \vec{p}^{n+1} \} + \frac{1}{\Delta t} [B] \{ \vec{p}^{n+1} \} \\ = \frac{1}{\Delta t} [B] \{ \vec{p}^n \} + \{ \vec{Q} \} \end{aligned}$$

$$\left[(1-\theta) [T] + \frac{1}{\Delta t} [B] \right] \{ \vec{p}^{n+1} \} = \left(\frac{1}{\Delta t} [B] - \theta [T] \right) \{ \vec{p}^n \} + \{ \vec{Q} \}$$

For $\theta = \frac{1}{2} \rightarrow$ Crank-Nicholson

$$\frac{\partial p}{\partial t} = \frac{p^{n+1} - p^{n-1}}{2\Delta t} + \underbrace{\theta (\Delta t)^2}_{\text{2nd order accurate}}$$