$$\frac{q_w}{q_w} = -\cancel{\frac{kd}{\mu B_\alpha}}\left(\cancel{4p_\ell} + \sum_{j=1}^{4}\left[\cancel{p_\ell} + \cancel{\frac{q_w \mu B_\alpha}{2\pi k d}}\ln\left(\frac{\Delta x}{r_{eq}}\right)\right] - \cancel{4p_\ell}\right)$$

$$1 = \frac{2}{\pi}\ln\left(\frac{\Delta x}{r_{eq}}\right) \Rightarrow \boxed{r_{eq} = \Delta x\, e^{-\frac{\pi}{2}} \approx 0.2078\,\Delta x \approx 0.2\,\Delta x}$$

"Peaceman correction"

If we have a rate constraint, i.e. q_w is fixed, then evaluate

p_w at r_w for $p_{ref} = p_\ell$ & $r_{req} = r_{eq}$

$$p_w = p_\ell + \underbrace{\frac{q_w \mu B_\alpha}{2\pi k d}\ln\left(\frac{0.2078\,\Delta x}{r_w}\right)}_{= 1/J_\ell^w} = p_\ell + \frac{q_w}{J_\ell^w}$$

If we have a pressure constraint, p_w is fixed, then solve for q_w

$$q^w = \frac{-2\pi k d}{\mu B_\alpha \ln\left(\frac{0.2078\,\Delta x}{r_w}\right)}\left(P_\ell - P_w\right) = -J_\ell^w\left(P_\ell - P_w\right)$$

where

skin factor

$$J_\ell^w = \frac{2\pi k d}{\mu B_\alpha\left[\ln\left(\frac{r_{eq}}{r_w}\right) + s\right]}$$

"productivity index" or "well index"

Going back

$$[B]\left\{\frac{\partial \vec{p}}{\partial t}\right\} + [T]\{\vec{P}\} = \{\vec{Q}\}$$

$$\{\vec{Q}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ q_l \\ 0 \\ 0 \\ 0 \end{Bmatrix} \longleftarrow \text{for a well in the } l^{th} \text{ grid block}$$

For a constant rate well

$$q_l = q_w$$

For a pressure constraint

$$q_l = -J_l\left(P_l - P_w\right)$$

$$[B]\left\{\frac{\partial p}{\partial t}\right\} + [T]\{\vec{P}\} + [J]\{\vec{P}\} = \{\vec{Q}\} = \begin{Bmatrix} 2TP_B \\ 0 \\ 0 \\ 0 \\ J_n P_w \\ 0 \\ 0 \\ 0 \\ q_m \end{Bmatrix}$$

prescribed pressure B.C.

constant BHP well in the n^{th} blk

constant rate well in the m^{th} blk.

$$[J] = \begin{bmatrix} 0 & & & & & & & & 0 \\ & 0 & & & & & & & \\ & & 0 & & & & & & \\ & & & 0 & & & & & \\ & & & & 0 & & & & \\ & & & & & J_n & & & \\ & & & & & & 0 & & \\ & & & & & & & 0 & \\ & 0 & & & & & & & 0 \; 0 \end{bmatrix} \leftarrow n^{th} \text{ grid blk}$$

Therefore for an <u>implicit</u> formulation

$$\underbrace{\left([T] + [J] + \frac{1}{\Delta t}[B]\right)}\left\{\vec{P}^{\,n+1}\right\} = \frac{1}{\Delta t}[B]\left\{\vec{P}^{\,n}\right\} + \left\{\vec{Q}\right\}$$