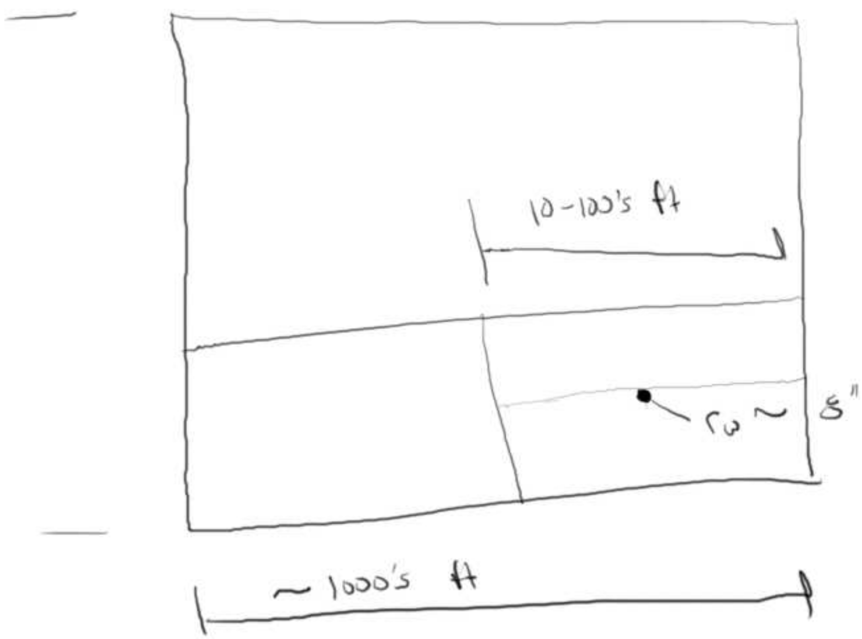
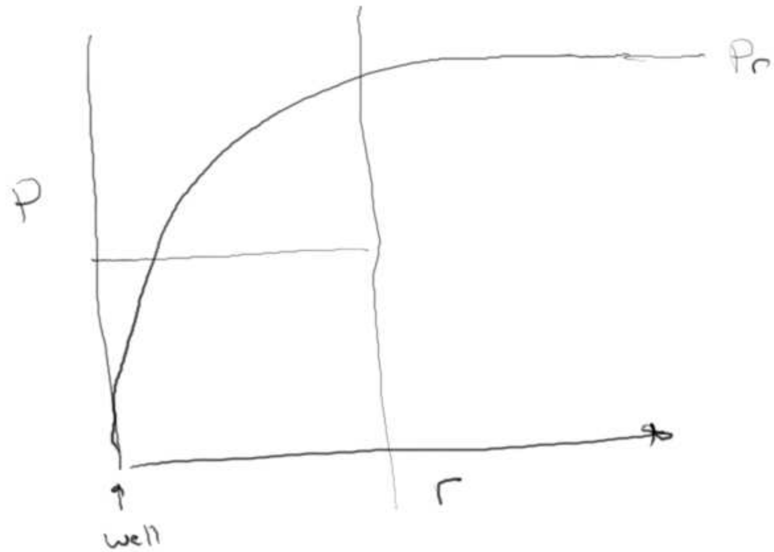
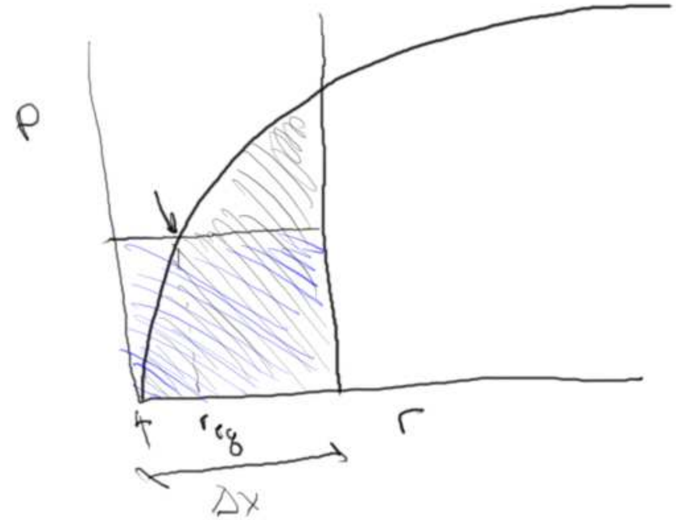
Well Models





Constraints
- Rate → need to compute p_w
- Pressure → need to compute q_w

Well models enrich the discrete solution with some information / knowledge of analytical solution:



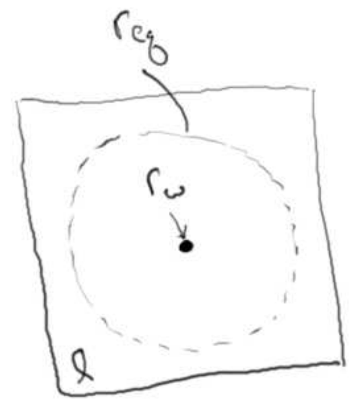
Radial diffusivity

assume s.s.

$$\frac{\phi \mu c_t}{k} \cancel{\frac{\partial p}{\partial t}} = \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{\partial p}{\partial r} \right)$$

$$\lim_{r \to 0} \left(r \frac{\partial p}{\partial r} \right) = \frac{-q_w \mu B_\alpha}{2\pi k d}$$

$$p = p_{ref} \left(r = r_{ref} \right)$$



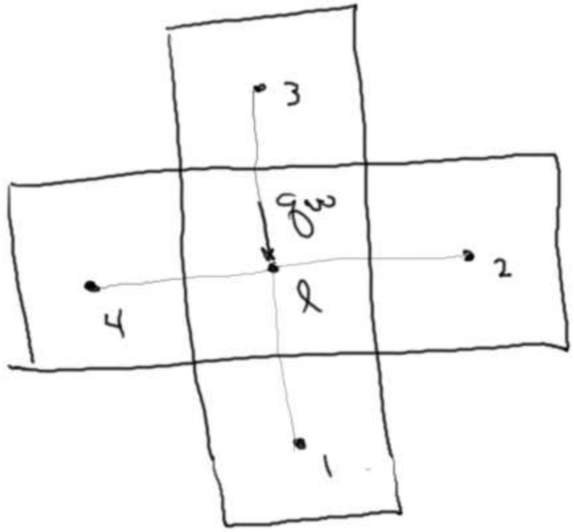
Solution:

$$p(r) = p_{ref} - \frac{q_w \mu B_\alpha}{2\pi k d} \ln \left(\frac{r}{r_{ref}} \right)$$

$$\downarrow$$

$$p_l = p_w - \frac{q_w \mu B_\alpha}{2\pi k d} \ln \left(\frac{r_{eq}}{r_w} \right)$$

Now consider



assume S.S.

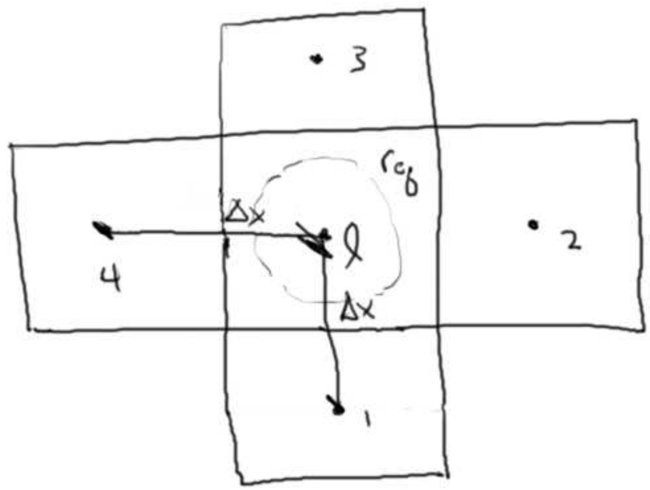
$$[B]\left\{\cancel{\frac{\partial P}{\partial t}}\right\} + [T]\{P\} = \{Q\}$$

$$T = \begin{bmatrix} -1 & & -1 & 2 & -1 & & -1 \end{bmatrix}$$

$$-\left\{\frac{k d \Delta x}{\mu B_o \Delta y}(P_1 - P_\ell) + \frac{k d \Delta y}{\mu B_o \Delta x}(P_2 - P_\ell) + \frac{k d \Delta x}{\mu B_o \Delta y}(P_3 - P_\ell) + \frac{k d \Delta y}{\mu B_o \Delta x}(P_4 - P_\ell)\right\} = q_w$$

if $\Delta x = \Delta y$

$$q_w = -\frac{kd}{\mu B_o}\left(P_1 + P_2 + P_3 + P_4 - 4P_\ell\right) \quad *$$



$$P(r) = P_{ref} - \frac{q_w \mu B_\alpha}{2\pi k d} \ln\left(\frac{r}{r_{ref}}\right)$$

evaluate at each grid block

w/ $P_{ref} = P_\ell$

$r_{ref} = r_{eq}$

$$\left.\begin{aligned}
P_1 &= P_\ell - \frac{q_w \mu B_\alpha}{2\pi k d} \ln\left(\frac{\Delta x}{r_{eq}}\right) \\
P_2 &= P_\ell - \frac{q_w \mu B_\alpha}{2\pi k d} \ln\left(\frac{\Delta x}{r_{eq}}\right) \\
P_3 &= \ldots \\
P_4 &= \ldots
\end{aligned}\right\} \text{Sub. in } *$$

$$\frac{q_w}{q_w} = -\frac{\cancel{kd}}{\cancel{\mu B_\alpha}}\left(\cancel{4p_\ell} + \sum_{j=1}^{4}\left[\cancel{p_\ell} + \frac{\cancel{q_w}\,\mu\,\cancel{B_\alpha}}{q_w\, 2\pi\, \cancel{kd}}\ln\left(\frac{\Delta x}{r_{eq}}\right)\right] - \cancel{4p_\ell}\right)$$

$$1 = \frac{2}{\pi}\ln\left(\frac{\Delta x}{r_{eq}}\right) \Rightarrow \boxed{r_{eq} = \Delta x\, e^{-\frac{\pi}{2}} \approx 0.2078\,\Delta x \approx 0.2\,\Delta x}$$

"Peaceman correction"

If we have a rate constraint, i.e. q_w is fixed, then evaluate p_w at r_w for $p_{ref} = p_\ell$ & $r_{req} = r_{eq}$

$$p_w = p_\ell + \underbrace{\frac{q_w\,\mu\,B_\alpha}{2\pi k d}\ln\left(\frac{0.2078\,\Delta x}{r_w}\right)}_{= 1/J_\ell^w} = p_\ell + \frac{q_w}{J_\ell^w}$$