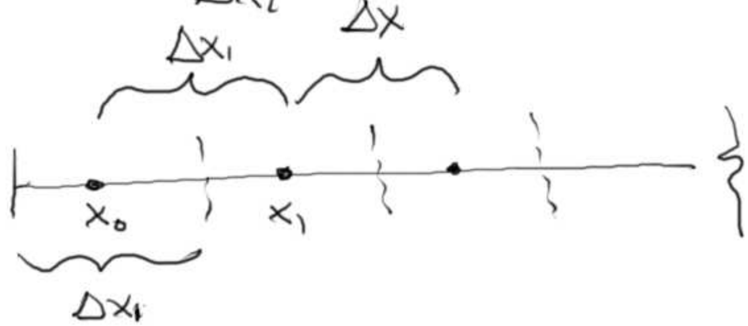
$$\sum_{i=0}^{N-1}\left[\frac{\partial p(x_i,t)}{\partial t} - \alpha \frac{\partial p^2(x_i,t)}{\partial x^2}\right] = 0$$

$$\frac{\partial^2 p(x)}{\partial x} = \frac{p(x_i+\Delta x_i) - 2p(x_i) + p(x_i-\Delta x_i)}{\Delta x_i^{\,2}} = \frac{p(x_{i+1}) - 2p(x_i) + p(x_{i-1})}{\Delta x^2}$$



$$\approx \boxed{\frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta x^2}}$$

$$\frac{\partial p_0}{\partial t} + \frac{\alpha}{\Delta x^2}\left[-p_1 + \underbrace{2p_0 - p_{-1}}\right] + \frac{\partial p_1}{\partial t} + \frac{\alpha}{\Delta x^2}\left[-p_2 + 2p_1 - p_0\right]$$

$$+ \dots + \frac{\partial p_{N-2}}{\partial t} + \frac{\alpha}{\Delta x^2}\left[-p_{N-1} + 2p_{N-2} - p_{N-3}\right]$$

$$+ \frac{\partial p_{N-1}}{\partial t} + \frac{\alpha}{\Delta x^2}\left[-p_N + 2p_{N-1} - p_{N-2}\right] = 0$$

$2p_0 - p_{-1}$

$p_0\left(2 - \frac{p_{-1}}{p_0}\right)$

$$\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}}_{[I]} \underbrace{\begin{Bmatrix} \frac{\partial p_0}{\partial t} \\ \frac{\partial p_1}{\partial t} \\ \vdots \\ \frac{\partial p_{N-1}}{\partial t} \end{Bmatrix}}_{\frac{\partial \vec{p}}{\partial t}} + \underbrace{\begin{bmatrix} 2-\frac{p_{-1}}{p_0} & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & 2 & -1 \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ & & -1 & 2 & -1 \\ 0 & \cdots & & -1 & 2-\frac{p_N}{p_{N-1}} \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_{N-2} \\ p_{N-1} \end{Bmatrix}}_{\vec{p}} = \vec{0}$$

$$[I]\left\{\frac{\partial\vec{p}}{\partial t}\right\} + \frac{\alpha}{\Delta x^2}[A]\{\vec{p}\} = \{\vec{0}\}$$

$$\left\{\frac{\partial\vec{p}}{\partial t}\right\} = -\frac{\alpha}{\Delta x^2}[I]^{-1}[A]\{\vec{p}\}$$

$$\underbrace{\left\{\frac{\partial\vec{p}}{\partial t}\right\} = -\frac{\alpha}{\Delta x^2}[A]\{\vec{p}\}}_{\text{ODE}} \Rightarrow \underbrace{\{\vec{p}\} = \exp\left(-\frac{\alpha}{\Delta x^2}[A]\right)\{\vec{p}_0\}}_{\text{solution}}$$

Forward-diff in t

$$\frac{\partial\vec{p}}{\partial t} = \frac{\vec{p}(t^n+\Delta t) - \vec{p}(t^n)}{\Delta t} = \frac{\vec{p}^{\,n+1} - \vec{p}^{\,n}}{\Delta t}$$