$$[I]\left\{\frac{\partial \vec{p}}{\partial t}\right\} + \frac{\alpha}{\Delta x^2}[A]\{\vec{p}\} = \{\vec{0}\}$$

$$\left\{\frac{\partial \vec{p}}{\partial t}\right\} = -\frac{\alpha}{\Delta x^2}[I]^{-1}[A]\{\vec{p}\}$$

$$\underbrace{\left\{\frac{\partial \vec{p}}{\partial t}\right\} = -\frac{\alpha}{\Delta x^2}[A]\{\vec{p}\}}_{\text{ODE}} \Rightarrow \underbrace{\{\vec{p}\} = \exp\left(-\frac{\alpha}{\Delta x^2}[A]\right)\{\vec{p}_0\}}_{\text{solution}}$$

Forward-diff in t

$$\frac{\partial \vec{p}}{\partial t} = \frac{\vec{p}(t^n + \Delta t) - \vec{p}(t^n)}{\Delta t} = \frac{\vec{p}^{n+1} - \vec{p}^n}{\Delta t}$$

$$[I]\{\vec{p}^{n+1} - \vec{p}^{n}\} + \frac{\alpha \Delta t}{\Delta x^2}[A]\{\vec{p}\} = \{\vec{0}\}$$

Evaluate $\vec{p}$ @ $\vec{p}^{n}$

$$[I]\{\vec{p}^{n+1}\} - [I]\{\vec{p}^{n}\} + \frac{\alpha \Delta t}{\Delta x^2}[A]\{\vec{p}^{n}\} = \{\vec{0}\}$$

$$\{\vec{p}^{n+1}\} = \left([I] - \frac{\alpha \Delta t}{\Delta x^2}[A]\right)\{\vec{p}^{n}\} \Leftarrow \text{Explicit}$$

$$\frac{\alpha \Delta t}{\Delta x^2} \leq 0.5$$

Evaluate $\vec{p}$ @ $\vec{p}^{n+1}$

$$[I]\{\vec{p}^{n+1}\} - [I]\{\vec{p}^{n}\} + \frac{\alpha \Delta t}{\Delta x^2}[A]\{\vec{p}^{n+1}\} = \{\vec{0}\}$$

$$\{\vec{p}^{n+1}\} = \left([I] + \frac{\alpha \Delta t}{\Delta x^2}[A]\right)^{-1}\{\vec{p}^{n}\} \Leftarrow \text{Implicit Method}$$