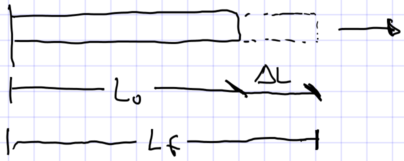


$$\varepsilon_{ENG} = \frac{\Delta L}{L_o} = \frac{\overbrace{L_f}^{\lambda} - L_o}{L_o} = \lambda - 1 \Rightarrow \lambda = \varepsilon_{ENG} + 1$$

Engineering strain → Lagrangian

$$\varepsilon_{LOG} = \int_{L_o}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_o}\right) = \ln(\lambda) = \ln(\varepsilon_{ENG} + 1)$$

Logrithmic strain, natural strain, "~~true strain~~"

$$\varepsilon_{TR} = \frac{L_f - L_o}{L_f} = 1 - \frac{1}{\lambda}$$

Eulerian strain, "~~true strain~~"

Seth-Hill strain

$$\varepsilon_{(m)} = \frac{1}{m}\left(\lambda^m - 1\right)$$

$m = 1$ → Eng. strain
$m = -1$ → Eulerian strain
$m = 0$ → Log strain

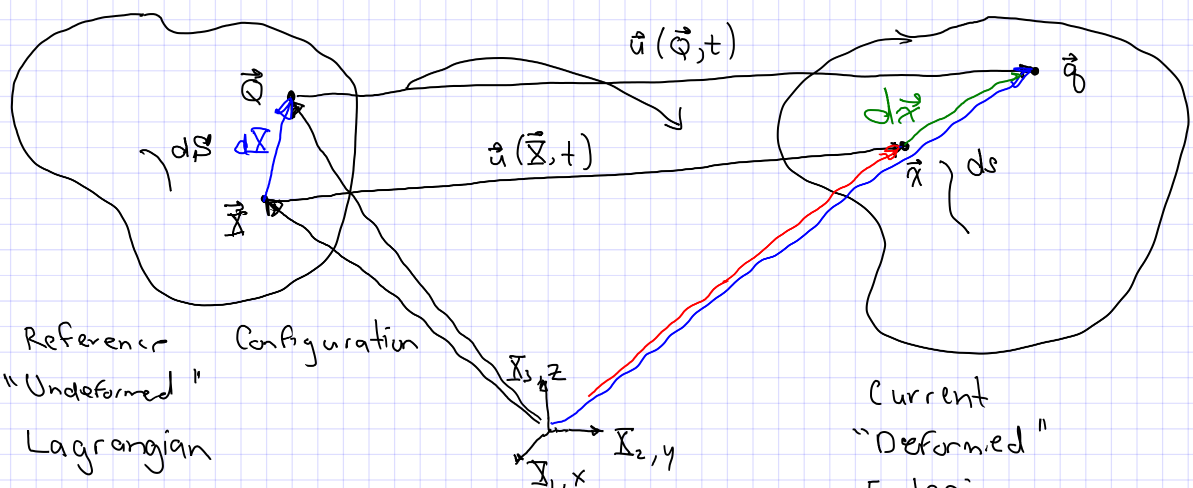
<u>Aside</u>

Please "label your strain"

Consider $\lambda = 1.01000$

$\varepsilon_{ENG} = \lambda - 1 = 0.01000$

$\varepsilon_{LOG} = \ln(\lambda) = 0.00990$



$$\vec{x} = \vec{X} + \vec{u}(\vec{X}, t)$$

$$\vec{q} = \vec{Q} + \vec{u}(\vec{Q}, t)$$

$$\vec{q} - \vec{x} = \vec{u}(\vec{Q}) - \vec{u}(\vec{X}) + (\vec{Q} - \vec{X})$$

Taylor expasion about $\vec{Q} = \vec{X}$

$$q_i - x_i = Q_i - X_i + \frac{\partial u_i}{\partial X_j}(Q_j - X_j) + \frac{\partial^2 u_i}{\partial X_j \partial X_k}(Q_j - X_j)(Q_k - X_k) + \text{H.O.T.'s}$$

$a = \{a_1, a_2, a_3\}$

$b = \{b_1, b_2, b_3\}$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 = \sum_{i=1}^{3} a_i b_i = a_i b_i$$

In indicial notation, repeated indices in "terms" imply summation

$$q_i - x_i = \delta_{ij}\left(Q_j - X_j\right) + \frac{\partial u_i}{\partial X_j}\left(Q_j - X_j\right) + \text{H.O.T.'s}$$

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases} \qquad a_i = \delta_{ij}\, a_j$$

$$\underbrace{q_i - x_i}_{dx_i} = \left(\delta_{ij} + \frac{\partial u_i}{\partial X_j}\right)\underbrace{\left(Q_j - X_j\right)}_{dX_j} + \cancel{O\left(\|\underbrace{\vec{Q} - \vec{X}}_{d\vec{X}}\|\right)}$$

$$dx_i = \left(\delta_{ij} + \frac{\partial u_i}{\partial X_j}\right) dX_j$$

$$d\vec{x} = \underbrace{\left(I + (\nabla_X \vec{u})^T\right)}_{F} d\vec{X}$$

$$d\vec{x} = F\, d\vec{X} \rightarrow \text{deformation gradient}$$