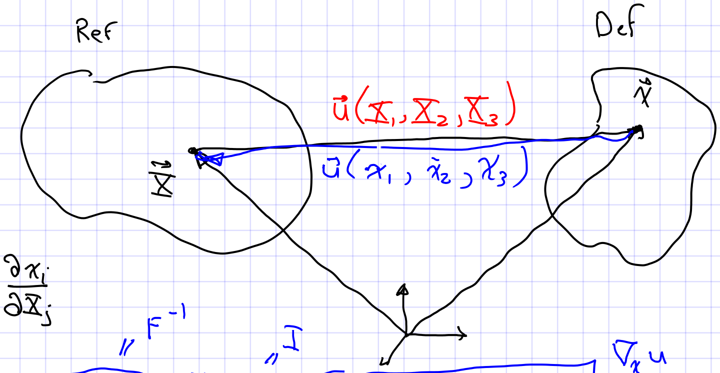
$$e = \frac{1}{2}\left[I - F^{-T}F^{-1} \right]$$

$$= \frac{1}{2}\left[\nabla_{\vec{x}}\vec{u} + (\nabla_{\vec{x}}\vec{u})^T + \cancelto{0}{(\nabla_{x}\vec{u})^T(\nabla_{x}\vec{u})} \right]$$

$$= \frac{1}{2}\left[\nabla_{\vec{x}}\vec{u} + (\nabla_{x}\vec{u})^T \right]$$

$$= \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \quad ///$$

$$F = \frac{\partial x_i}{\partial X_j}$$

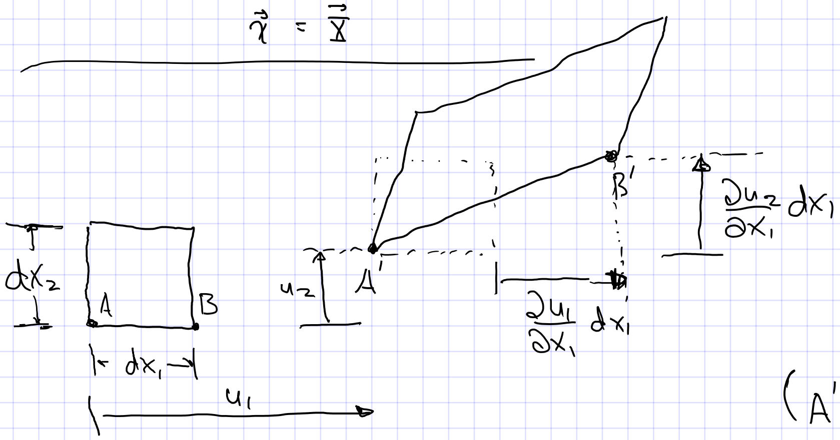


$$\underbrace{\boxed{\frac{\partial \vec{X}}{\partial \vec{x}}}}_{= F^{-1}} = \underbrace{\boxed{\frac{\partial \vec{x}}{\partial \vec{x}}}}_{= I} + \underbrace{\boxed{\frac{\partial \vec{u}(x_1, x_2, x_3)}{\partial \vec{x}}}}_{\nabla_x u}$$

$$F^{-1} = I + \nabla_{\vec{x}}\vec{u}$$

$$\vec{x} = \vec{X} + \vec{u}(\vec{X})$$

$$\vec{x} = \vec{X}$$



$$\varepsilon_{(x_1)} = \frac{A'B' - AB}{AB}$$

$$= \frac{A'B' - dx_1}{dx_1} = \frac{A'B'}{dx_1} - 1$$

$$(A'B')^2 = \left[\left(dx_1 + \frac{\partial u_1}{\partial x_1} dx_1\right)^2 + \left(\frac{\partial u_2}{\partial x_1} dx_1\right)^2 \right]$$

$$\varepsilon_{(x_1)}^2 = \left(\frac{A'B'}{dx_1} - 1\right)\left(\frac{A'B'}{dx_1} - 1\right)$$

$$\cancelto{0}{\varepsilon_{(x_1)}^2} = \cancel{2}\varepsilon_{(x_1)} + \cancel{1} = \cancel{1} + \cancel{2}\frac{\partial u_1}{\partial x_1} + \cancelto{}{\left(\frac{\partial u_1}{\partial x_1}\right)^2} + \cancelto{0}{\left(\frac{\partial u_2}{\partial x_1}\right)^2}$$

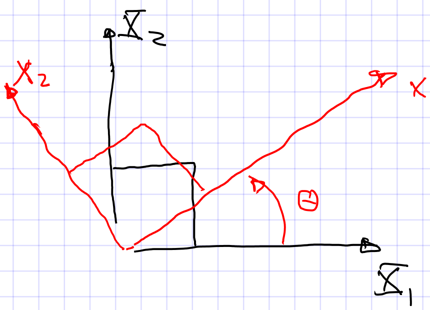
$$\varepsilon_{(x_1)} = \frac{\partial u_1}{\partial x_1}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\varepsilon_{11} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\right)$$

$$= \frac{\cancel{1}}{\cancel{2}}\left(\cancel{2}\frac{\partial u_1}{\partial x_1}\right)$$

$$= \frac{\partial u_1}{\partial x_1}$$



$$\begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{Bmatrix} = \overbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{R} \begin{Bmatrix} dX_1 \\ dX_2 \\ dX_3 \end{Bmatrix}$$

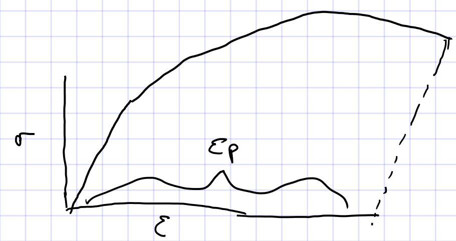
$$d\vec{x} = F \, d\vec{X} \qquad F = R$$

$$E = \frac{1}{2}\left[F^T F - I\right] = \frac{1}{2}\left[R^T R - I\right]$$
$$= \frac{1}{2}\left[R^{-1} R - I\right]$$
$$= \frac{1}{2}\left[I - I\right]$$
$$= \underline{\underline{0}}$$

$$F = \nabla u^T + I$$
$$\nabla u^T = F - I$$

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$
$$= \frac{1}{2}\left(F - I + (F - I)^T\right) = \frac{1}{2}\left[F + F^T\right] - I$$
$$= \frac{1}{2}\left[R + R^T\right] - I$$
$$\neq \underline{\underline{0}} \quad \text{for any } R \neq I$$

$$\sigma = E\,\varepsilon$$



Strain-rate

$$\frac{d}{dt}\left(d\vec{x}\right) = d\left(\overset{\vec{v}}{\frac{\partial \vec{x}}{\partial t}}\right) = d\vec{v}$$

$$\vec{v} = \vec{v}\left(x_1, x_2\, x_3, t\right)$$

$$dv_i = \underbrace{\frac{\partial v_i}{\partial x_j}}_{L\ =\ \text{velocity gradient}} dx_j$$

$$\frac{d}{dt}\left(F_{ij}\right) = \frac{d}{dt}\left(\frac{\partial x_i}{\partial X_j}\right) = \frac{\partial}{\partial X_j}\left(\overset{v_i}{\frac{\partial x_i}{\partial t}}\right)$$

$$\dot{F}_{ij}\frac{\partial X_j}{\partial x_k} = \frac{\partial v_i}{\cancel{\partial X_j}}\frac{\cancel{\partial X_j}}{\partial x_k} = \frac{\partial v_i}{\partial x_k}$$

$$\boxed{\dot{F} F^{-1} = L}$$

$$\frac{d}{dt}\left[(ds)^2 - \overset{0}{\cancel{(dS)^2}}\right] = \frac{d}{dt}\left[d\vec{X}^T (2E)\, d\vec{X}\right] = \frac{d}{dt}\left[d\vec{x}^T d\vec{x}^T - \overset{0}{\cancel{d\vec{X}^T d\vec{X}}}\right]$$

$$\frac{d}{dt}(ds)^2 = \frac{d}{dt}\left(d\vec{x}^T d\vec{x}\right)$$
$$= d\left(\overset{\vec{v}^T}{\frac{d\vec{x}}{dt}}\right) d\vec{x} + d\vec{x}^T d\left(\overset{\vec{v}}{\frac{d\vec{x}}{dt}}\right)$$
$$= d\vec{v}^T d\vec{x} + d\vec{x}^T d\vec{v}$$
$$= \left(L\, d\vec{x}\right)^T d\vec{x} + d\vec{x}^T\left(L\, d\vec{x}\right)$$
$$= d\vec{x}^T L^T d\vec{x} + d\vec{x}^T L\, d\vec{x}$$
$$= d\vec{x}^T \underbrace{\left(L^T + L\right)}_{2D} d\vec{x}$$

$$\boxed{D = \frac{1}{2}\left(L^T + L\right)}$$

D → rate-of-deformation

$$D = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$$\dot{\varepsilon} = \frac{1}{2}\left(\frac{\partial \dot{u}_i}{\partial X_j} + \frac{\partial \dot{u}_j}{\partial X_i}\right)$$
$$= \frac{1}{2}\left(\frac{\partial v_i}{\partial X_j} + \frac{\partial v_j}{\partial X_i}\right)$$

$$\vec{x} = \vec{X} + \vec{u}$$