$$\frac{d}{dt}(ds)^2 = d\vec{X}^T \left(\boxed{2 \frac{dE}{dt}} \right) d\vec{X}$$

$$\boxed{d\vec{x} = F\, d\vec{X}}$$

$$\frac{d}{dt}(ds)^2 = d\vec{x}^T (2D)\, d\vec{x} \quad *$$

$$= (F\, d\vec{X})^T (2D)(F\, d\vec{X})$$

$$= d\vec{X}^T \boxed{F^T (2D)\, F}\, d\vec{X}$$

$$\frac{d\vec{x}^T (\cancel{2} D)\, d\vec{x}}{\cancel{2}} = \frac{\cancel{2}\, d\vec{x}^T (L^T e + \dot{e} + eL)\, d\vec{x}}{\cancel{2}}$$

$$D = L^T e + \dot{e} + eL$$

$$\boxed{\frac{dE}{dt} = F^T D F} \rightarrow \text{rate-of-green strain}$$

Eulerian strain rate

$$\frac{d}{dt}(ds)^2 = \frac{d}{dt}\left(d\vec{x}^T (2e)\, d\vec{x} \right)$$

$$= 2\left[(L\, d\vec{x})^T e\, d\vec{x} + d\vec{x}^T \frac{de}{dt} d\vec{x} + d\vec{x}^T e\, (L\, d\vec{x}) \right]$$

$$= \cancel{2}\, d\vec{x}^T \underbrace{(L^T e + \dot{e} + eL)}_{\cancel{2} D} d\vec{x} \quad **$$

$$\boxed{\dot{e} = D - L^T e - eL} \rightarrow \text{rate-of-Eulerian strain}$$

Stress (Heuristic argument)

$$\vec{x} = \vec{x}(\vec{X}, t) \rightarrow \text{"current" configuration}$$

$$P(1) = \frac{dW}{dt} = \int \vec{f} \cdot \cancel{\frac{d\vec{x}}{dt}}\, \vec{v} \left(\frac{dV}{dV} \right)$$

$$dV = \vec{A}\, d\vec{x} = A_j\, dx_j$$

$$= \int \frac{f_i}{A_j} \frac{dv_i}{dx_j}\, dV$$

$$L = \text{symm}(L) + \text{antisymm}(L)$$

$$= \underbrace{\frac{1}{2}(L^T + L)}_{D} + \underbrace{\frac{1}{2}(L - L^T)}_{W}$$

$$\int \sigma : L\, dV$$

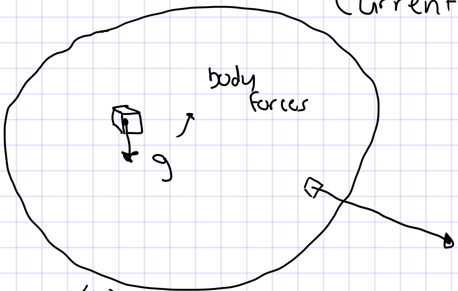
$$\sigma : L = \sigma_{ij} L_{ij}$$

$$\sigma : W = \bar{\bar{0}}$$

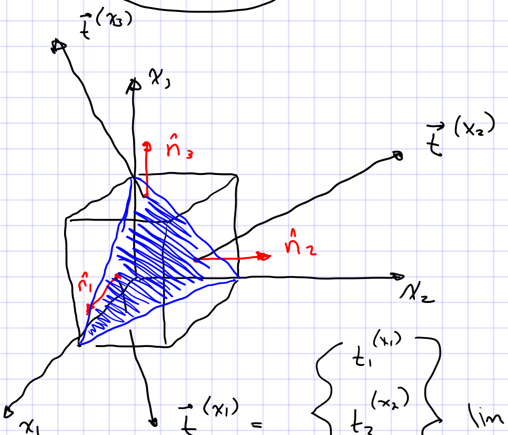
$$\int \sigma : (D + W)\, dV$$

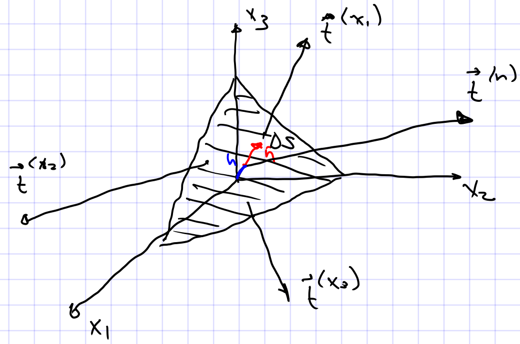
$$P = \int \sigma : D\, dV$$

Current



surface forces → tractions → $\frac{\text{force}}{\text{area}}$





$$\hat n = \begin{Bmatrix} \cos(\hat n, x_1) \to n_1 \\ \cos(\hat n, x_2) \to n_2 \\ \cos(\hat n, x_3) \to n_3 \end{Bmatrix}$$

$$\vec t^{(x_1)} = \begin{Bmatrix} t_1^{(x_1)} \\ t_2^{(x_2)} \\ t_3^{(x_3)} \end{Bmatrix}$$

$$\lim_{h\to 0} \sum F = \vec t^{(n)} \cancel{\Delta S} - \vec t^{(x_2)} n_2 \cancel{\Delta S} - \vec t^{(x_3)} n_3 \cancel{\Delta S} - \vec t^{(x_1)} n_1 \cancel{\Delta S} = \cancel{\rho \tfrac{1}{3} h \Delta S \vec a}^{\,0}$$

$$\vec t^{(n)} = \vec t^{(x_1)} n_1 + \vec t^{(x_2)} n_2 + \vec t^{(x_3)} n_3$$

$$\vec t^{(n)T} = \vec t^{(x_1)T} n_1 + \vec t^{(x_2)T} n_2 + \vec t^{(x_3)T} n_3$$

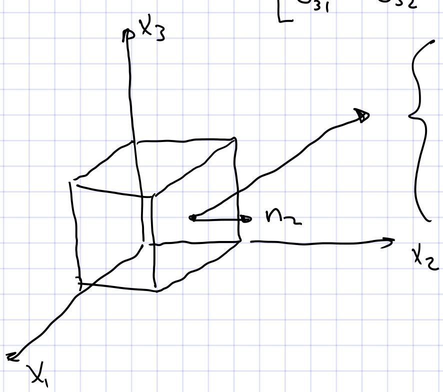
$$\vec t^{(n)T} = [n_1 \; n_2 \; n_3] \underbrace{\begin{bmatrix} \vec t^{(x_1)T} \\ \vec t^{(x_2)T} \\ \vec t^{(x_3)T} \end{bmatrix}}_{\text{Cauchy stress } \sigma}$$

$$\boxed{\vec t^{(n)T} = \hat n^T \sigma}$$

$$\boxed{\vec t = \sigma^T \hat n} \to \text{Cauchy stress eqn.}$$

$$t_i = \sigma_{ji} n_j$$

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$



$$\begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{Bmatrix}$$