



$$\vec{u}' = R \vec{u}$$

$$\vec{v}' = R \vec{v}$$

$$\vec{v} = T \vec{u}$$

$$\rightarrow \vec{v}' = T' \vec{u}'$$

$$R \vec{v} = R T \vec{u} = R T R^{-1} R \vec{u} = R T R^{-1} R \vec{u}$$

$$\vec{v} = R^{-1} T' R \vec{u}$$

$$\vec{v} = R^{-1} T' R \vec{u}$$

$$T = R^{-1} T' R$$

$$T' = R T R^{-1}$$

$$\sigma' = R \sigma R^{-1} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix}$$

$$\sigma_I > \sigma_{II} > \sigma_{III}$$

$$\det(\sigma - \lambda I) = 0$$

$$\lambda_1, \lambda_2, \lambda_3$$

$$(\sigma - \lambda_1 I) \vec{v}_1 = \vec{0}$$

$$(\sigma - \lambda_2 I) \vec{v}_2 = \vec{0}$$

$$(\sigma - \lambda_3 I) \vec{v}_3 = \vec{0}$$

$$A = Q \Lambda Q^T$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{bmatrix}$$

$$R = \left[ \frac{\vec{v}_1}{\|\vec{v}_1\|}, \frac{\vec{v}_2}{\|\vec{v}_2\|}, \frac{\vec{v}_3}{\|\vec{v}_3\|} \right]$$

$$-\lambda^3 + I_1 \lambda^2 + I_2 \lambda + I_3 = 0 \leftarrow \text{characteristic equation}$$

$I_1, I_2, I_3$  are invariants of the stress tensor

$$I_1 = \text{tr}(\sigma) = \sigma_{ii} = \underbrace{\sigma_I + \sigma_{II} + \sigma_{III}}_{\exists p}$$

$$I_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{2} I_1^2$$

$$= -(\sigma_I \sigma_{II} + \sigma_I \sigma_{III} + \sigma_{II} \sigma_{III})$$

$$I_3 = \det(\sigma) = \underbrace{\sigma_I \sigma_{II} \sigma_{III}}_{\text{pressure}}$$

$$\sigma_{ij} = \underbrace{-\frac{1}{3} \sigma_{kk} \delta_{ij}}_{\text{spherical hydrostatic dilational}} + \underbrace{S_{ij}}_{\text{deviatoric stress}}$$

spherical hydrostatic dilational

deviatoric stress

$$\rho \ddot{u} = \nabla \cdot \sigma + \rho b$$

$$\sigma \Rightarrow f(u)$$

"constitutive model"

$$\sigma = E \epsilon = E \left( \frac{\partial u}{\partial x} \right)$$

$$U S \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$$

$$A = L U$$

Invariants of deviatoric stress

$$J_1 = 0$$

$$J_2 = \frac{1}{2} S_{ij} S_{ij}$$

$$J_3 = \det(s)$$