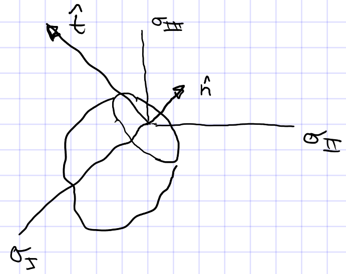


$$\vec t = \sigma^T \hat n = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} \sigma_I n_1 \\ \sigma_{II} n_2 \\ \sigma_{III} n_3 \end{Bmatrix}$$

$$t_n = \hat t \cdot \hat n = \sigma_I n_1^2 + \sigma_{II} n_2^2 + \sigma_{III} n_3^2 \quad - \; 1$$

$$|t|^2 = t_s^2 + t_n^2$$

$$t^2 = t_n^2 + t_s^2 = (\sigma_I \hat n_1)^2 + (\sigma_{II} n_2)^2 + (\sigma_{III} n_3)^2 \quad - \; 2$$

$$|n| = 1 = n_1^2 + n_2^2 + n_3^2 \quad - \; 3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ \sigma_I & \sigma_{II} & \sigma_{III} \\ \sigma_I^2 & \sigma_{II}^2 & \sigma_{III}^2 \end{bmatrix} \begin{Bmatrix} n_1^2 \\ n_2^2 \\ n_3^2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ t_n \\ t_s^2 + t_n^2 \end{Bmatrix}$$

$$n_1^2 = \frac{t_s^2 + (t_n - \sigma_{II})(t_n - \sigma_{III})}{\underbrace{(\sigma_I - \sigma_{II})(\sigma_I - \sigma_{III})}_{\geq 0}} \geq 0$$

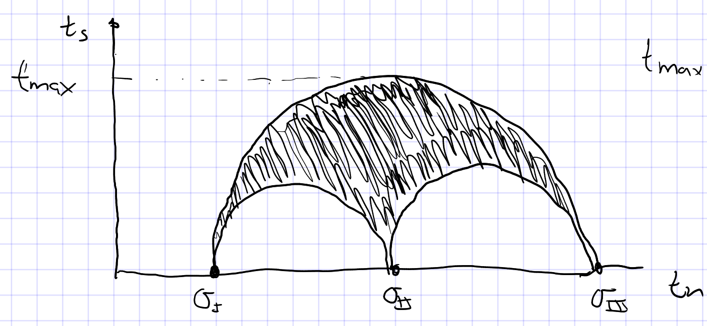
$$n_2^2 = \frac{t_s^2 + (t_n - \sigma_{III})(t_n - \sigma_I)}{\underbrace{(\sigma_{II} - \sigma_{III})(\sigma_{II} - \sigma_I)}_{\leq 0}} \leq 0$$

$$n_3^2 = \frac{t_s^2 + (t_n - \sigma_I)(t_n - \sigma_{II})}{\underbrace{(\sigma_I - \sigma_{III})(\sigma_{II} - \sigma_{III})}_{\geq 0}} \geq 0$$

$$\underbrace{\left[t_n - \tfrac{1}{2}(\sigma_{II} + \sigma_{III})\right]^2}_{x^2} + \underbrace{t_s^2}_{y^2} \geq \underbrace{\left(\tfrac{1}{2}(\sigma_{II} - \sigma_{III})\right)^2}_{r^2}$$

$$\left[t_n - \tfrac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\tfrac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$

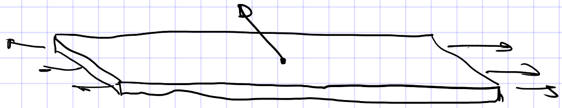
$$\left[t_n - \tfrac{1}{2}(\sigma_I + \sigma_{II})\right]^2 + t_s^2 \leq \left(\tfrac{1}{2}(\sigma_I - \sigma_{II})\right)^2$$

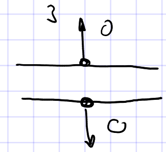


$$t_{max} = \tfrac{1}{2}(\sigma_I + \sigma_{III})$$

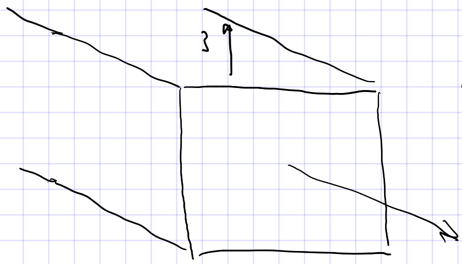
Mohr's circles

Plane stress

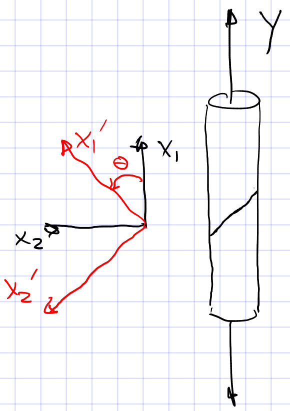




Plane strain



$\varepsilon_{33} \approx 0$



$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

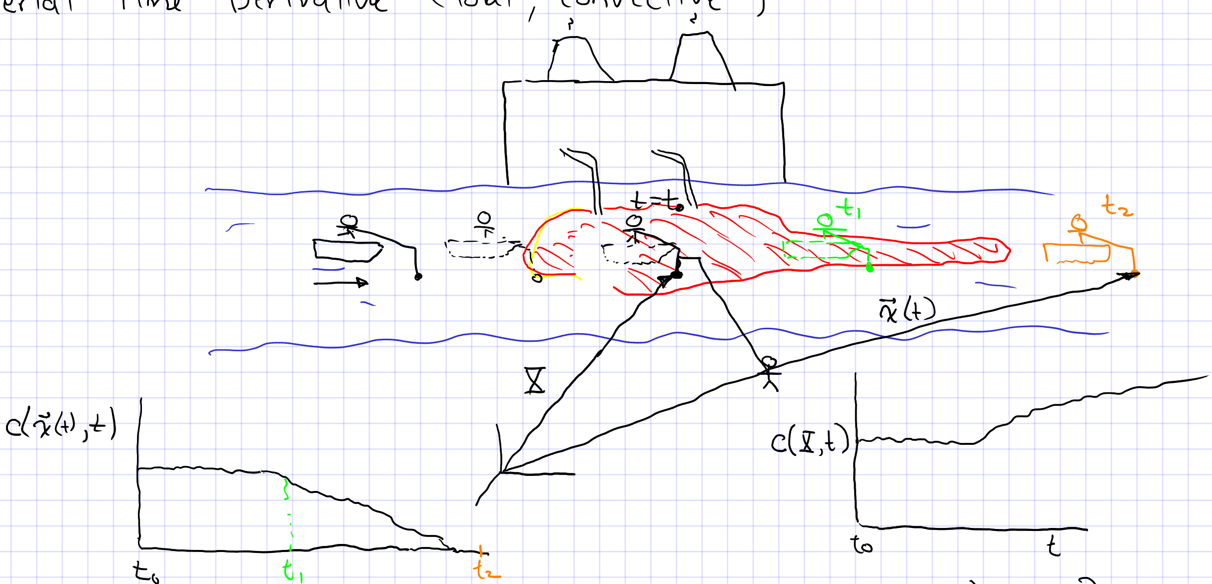
$$\sigma' = R \sigma R^T \qquad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_{11}' = Y \cos\theta$$

$$\sigma_{12}' = \frac{Y}{2} \sin\theta \cos\theta - \frac{Y}{2} \sin 2\theta$$

max @ 45°

Material Time Derivative (total, convective)



$$\frac{D}{Dt}\left(c(\underline{X}, t)\right) = \frac{\partial c}{\partial t}$$

$$\frac{D}{Dt}\left(c(\vec{x}(t), t)\right) = \frac{\partial c}{\partial t} + \frac{\partial c}{\partial \vec{x}} \frac{\partial \vec{x}}{\partial t}$$

$$= \underbrace{\frac{\partial c}{\partial t}}_{\text{local rate-of-change}} + \underbrace{\nabla c \cdot \vec{v}}_{\text{convective rate-of-change}}$$

$$\boxed{\frac{D}{Dt}(\cdot) = \frac{\partial}{\partial t}(\cdot) + \nabla(\cdot) \cdot \vec{v}} \leftarrow \text{material time derivative}$$