In[7]:= **u[x_] = c1 x (1 – x) + c2 x^2 (1 – x) + c3 x^3 (1 – x)**

Out[7]= $c1\,(1-x)\,x + c2\,(1-x)\,x^2 + c3\,(1-x)\,x^3$

In[8]:= **u[0]**

Out[8]= 0

In[9]:= **u[1]**

Out[9]= 0

In[11]:= **II = $\int_0^1 (u'[x]^2 - u[x]^2 + 2\,x^2\,u[x])\,dx$**

Out[11]= $\frac{c1}{10} + \frac{3\,c1^2}{10} + \frac{c2}{15} + \frac{3\,c1\,c2}{10} + \frac{13\,c2^2}{105} + \frac{c3}{21} + \frac{19\,c1\,c3}{105} + \frac{79\,c2\,c3}{420} + \frac{103\,c3^2}{1260}$

In[13]:= **eqn1 = D[II, c1] == 0; eqn1**

Out[13]= $\frac{1}{10} + \frac{3\,c1}{5} + \frac{3\,c2}{10} + \frac{19\,c3}{105} == 0$

In[14]:= **eqn2 = D[II, c2] == 0; eqn2**

Out[14]= $\frac{1}{15} + \frac{3\,c1}{10} + \frac{26\,c2}{105} + \frac{79\,c3}{420} == 0$

In[17]:= **eqn3 = D[II, c3] == 0; eqn3**

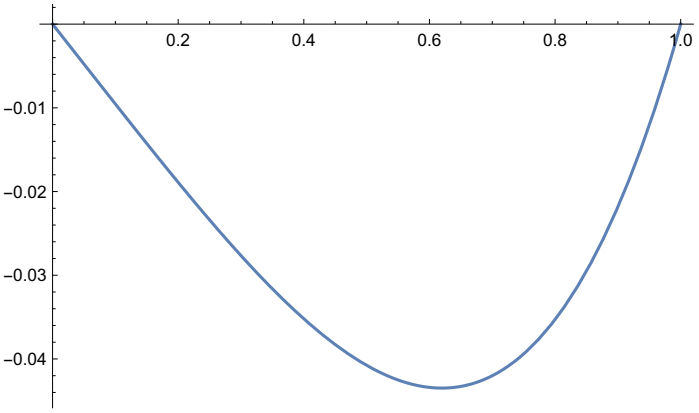
Out[17]= $\frac{1}{21} + \frac{19\,c1}{105} + \frac{79\,c2}{420} + \frac{103\,c3}{630} == 0$

In[19]:= **sol = First@Solve[{eqn1, eqn2, eqn3}, {c1, c2, c3}]**

Out[19]= $\left\{c1 \to -\frac{2335}{24\,518},\ c2 \to -\frac{1232}{12\,259},\ c3 \to -\frac{21}{299}\right\}$

In[21]:= **Plot[u[x] /. sol, {x, 0, 1}]**

Out[21]=



In[41]:= **u[x_] = c1 + c2 x + c3 x^2**

Out[41]= $c1 + c2\,x + c3\,x^2$

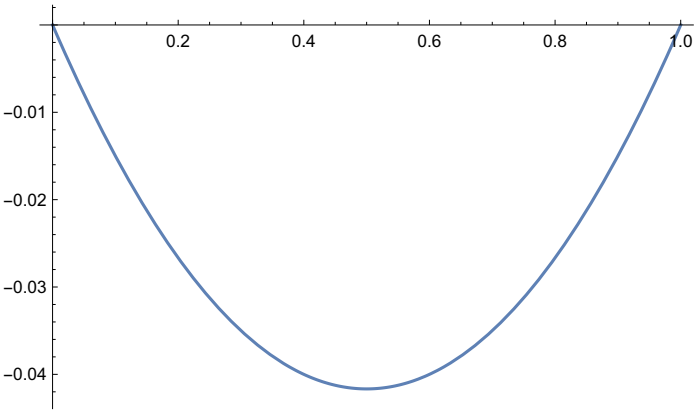
In[42]:=
```
bc1 = Solve[u[0] == 0, c1] // First
```
Out[42]=
$$\{c1 \to 0\}$$

In[43]:=
```
bc2 = Solve[(u[1] /. bc1) == 0, c2] // First
```
Out[43]=
$$\{c2 \to -c3\}$$

In[44]:=
```
II = ∫_0^1 (u'[x]^2 - u[x]^2 + 2 x^2 u[x]) dx /. bc1 /. bc2
```
Out[44]=
$$-\frac{c3}{10} + \frac{3\,c3^2}{10}$$

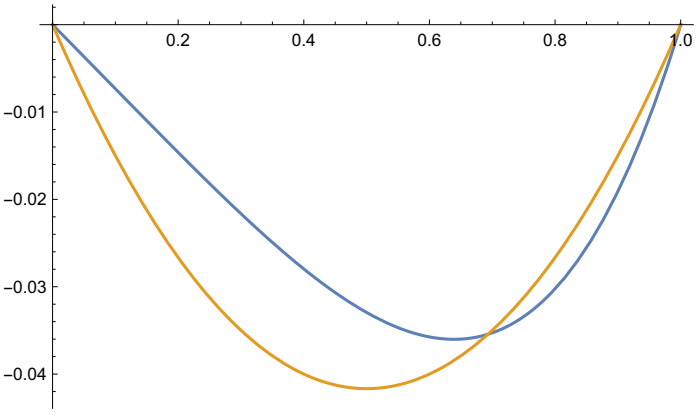
In[48]:=
```
sol = Solve[D[II, c3] == 0, c3] // First
```
Out[48]=
$$\left\{c3 \to \frac{1}{6}\right\}$$

In[55]:=
```
p1 = Plot[u[x] /. bc1 /. bc2 /. sol, {x, 0, 1}]
```
Out[55]=



In[53]:=
```
exact = First@DSolve[{-v''[x] + v[x] + x^2 == 0, v[0] == 0, v[1] == 0}, v[x], x]
```
Out[53]=
$$\{v[x] \to \left(e^{-x}\left(-3\,e + 2\,e^2 + 2\,e^x - 2\,e^{2\,x} - 2\,e^{2+x} + 3\,e^{1+2\,x} + e^x\,x^2 - e^{2+x}\,x^2\right)\right) / \left(-1 + e^2\right)\}$$

In[58]:=
```
p2 = Plot[{v[x] /. exact, u[x] /. bc1 /. bc2 /. sol}, {x, 0, 1}]
```
Out[58]=



In[60]:= **u[x_] = c1 + c2 x + c3 x^2 + c4 x^3**

Out[60]= $c1 + c2\,x + c3\,x^2 + c4\,x^3$

In[61]:= **bc1 = Solve[u[0] == 0, c1] // First**

Out[61]= $\{c1 \to 0\}$

In[62]:= **bc2 = Solve[(u[1] /. bc1) == 0, c2] // First**

Out[62]= $\{c2 \to -c3 - c4\}$

In[63]:= **II = $\int_0^1$ (u '[x]^2 - u[x]^2 + 2 x^2 u[x]) dx /. bc1 /. bc2**

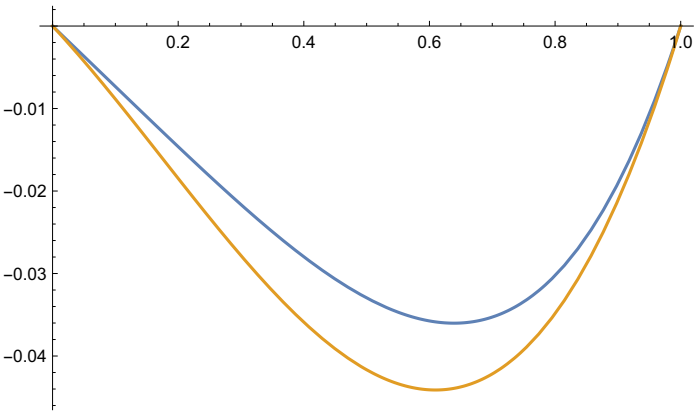
Out[63]= $\frac{2\,c3}{5} + \frac{17\,c3^2}{15} + \frac{1}{2}(-c3 - c4) + \frac{3}{2}\,c3\,(-c3 - c4) + \frac{2}{3}(-c3 - c4)^2 + \frac{c4}{3} + \frac{8\,c3\,c4}{3} + \frac{8}{5}(-c3 - c4)\,c4 + \frac{58\,c4^2}{35}$

In[64]:= **sol = Solve[{D[II, c3] == 0, D[II, c4] == 0}, {c3, c4}] // First**

Out[64]= $\left\{c3 \to -\frac{11}{123},\ c4 \to \frac{7}{41}\right\}$

In[66]:= **p2 = Plot[{v[x] /. exact, u[x] /. bc1 /. bc2 /. sol}, {x, 0, 1}]**

Out[66]=



In[67]:= **u[x_] = c1 + c2 x + c3 x^2 + c4 x^3 + c5 x^4**

Out[67]= $c1 + c2\,x + c3\,x^2 + c4\,x^3 + c5\,x^4$

In[68]:= **bc1 = Solve[u[0] == 0, c1] // First**

Out[68]= $\{c1 \to 0\}$

In[69]:= **bc2 = Solve[(u[1] /. bc1) == 0, c2] // First**

Out[69]= $\{c2 \to -c3 - c4 - c5\}$

In[70]:=
$$II = \int_0^1 \left(u'[x]^2 - u[x]^2 + 2\,x^2\,u[x]\right) dx \;/.\; bc1 \;/.\; bc2$$

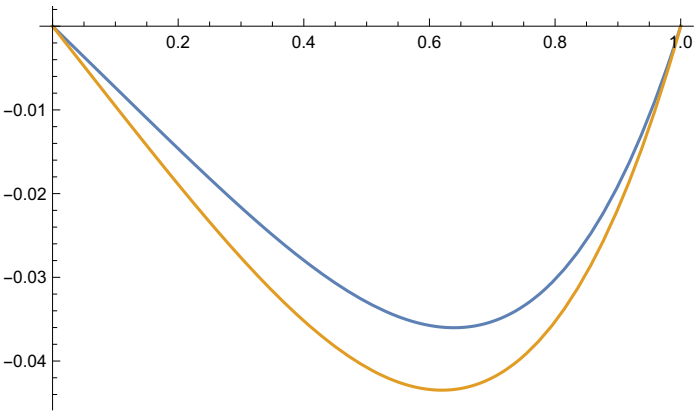
Out[70]=
$$\frac{2\,c3}{5} + \frac{17\,c3^2}{15} + \frac{c4}{3} + \frac{8\,c3\,c4}{3} + \frac{58\,c4^2}{35} + \frac{1}{2}(-c3-c4-c5) + \frac{3}{2}\,c3\,(-c3-c4-c5) + \frac{8}{5}\,c4\,(-c3-c4-c5) + \frac{2}{3}(-c3-c4-c5)^2 + \frac{2\,c5}{7} + \frac{102\,c3\,c5}{35} + \frac{15\,c4\,c5}{4} + \frac{5}{3}(-c3-c4-c5)\,c5 + \frac{137\,c5^2}{63}$$

In[71]:=
```
sol = Solve[{D[II, c3] == 0, D[II, c4] == 0, D[II, c5] == 0}, {c3, c4, c5}] // First
```

Out[71]=
$$\left\{c3 \to -\frac{129}{24\,518},\ c4 \to \frac{371}{12\,259},\ c5 \to \frac{21}{299}\right\}$$

In[72]:=
```
p2 = Plot[{v[x] /. exact, u[x] /. bc1 /. bc2 /. sol}, {x, 0, 1}]
```

Out[72]=



In[77]:=
```
X = {1, x, x^2, x^3}
```

Out[77]=
$$\{1,\ x,\ x^2,\ x^3\}$$

In[78]:=
```
A = {X /. {x → 0}, X /. {x → L / 3}, X /. {x → 2 L / 3}, X /. {x → L}}; MatrixForm[A]
```

Out[78]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{L}{3} & \frac{L^2}{9} & \frac{L^3}{27} \\ 1 & \frac{2\,L}{3} & \frac{4\,L^2}{9} & \frac{8\,L^3}{27} \\ 1 & L & L^2 & L^3 \end{pmatrix}$$

In[79]:=
```
NN = X.Inverse[A]
```

Out[79]=
$$\left\{1 - (11\,x)/(2\,L) + (9\,x^2)/L^2 - (9\,x^3)/(2\,L^3),\ (9\,x)/L - (45\,x^2)/(2\,L^2) + (27\,x^3)/(2\,L^3),\right.$$
$$\left. -((9\,x)/(2\,L)) + (18\,x^2)/L^2 - (27\,x^3)/(2\,L^3),\ \frac{x}{L} - (9\,x^2)/(2\,L^2) + (9\,x^3)/(2\,L^3)\right\}$$