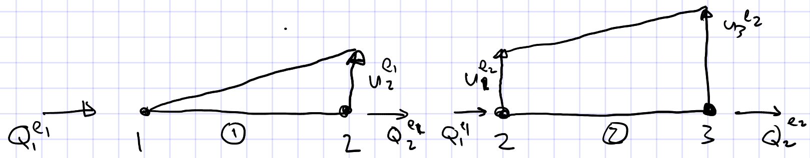


2n unknowns

$(u_1, u_2, \dots, u_n)$ & $(Q_1, Q_2, \dots, Q_n)$

Continuity in u_i's & "balance" Q_i's

$$Q_2^{e_1} + Q_1^{e_2} = \begin{cases} 0 & \text{if no external sources} \\ Q_J & \text{if external source of mag. } Q_J \end{cases}$$

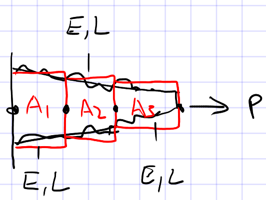
$$u_2^{e_1} = u_2^{e_2} = u_2$$

$$K^{e_1} = \begin{bmatrix} K_{11}^1 & K_{12}^1 \\ K_{21}^1 & K_{22}^1 \end{bmatrix}, \quad K^{e_2} = \begin{bmatrix} K_{11}^2 & K_{12}^2 \\ K_{21}^2 & K_{22}^2 \end{bmatrix} \qquad K^G = \begin{bmatrix} K_{11}^1 & K_{12}^1 & 0 \\ K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\ 0 & K_{21}^2 & K_{22}^2 \end{bmatrix}$$

$$\left[K^G \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} f_1^1 \\ f_2^1 + f_1^2 \\ f_2^2 \end{Bmatrix} + \begin{Bmatrix} Q_1^1 \\ Q_2^1 + Q_1^2 \\ Q_2^2 \end{Bmatrix}$$

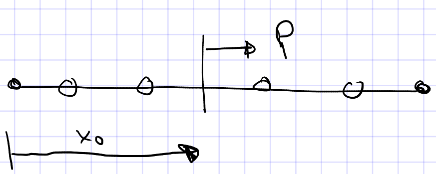
$$f_J = \int f(x) N_J \, dx$$

Ex



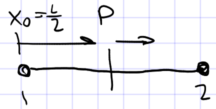
$$\begin{bmatrix} \frac{A_1 E}{L} & -\frac{A_1 E}{L} & 0 & 0 \\ -\frac{A_1 E}{L} & \frac{(A_1 + A_2)E}{L} & -\frac{A_2 E}{L} & 0 \\ 0 & -\frac{A_2 E}{L} & \frac{(A_2 + A_3)E}{L} & -\frac{A_3 E}{L} \\ 0 & 0 & -\frac{A_3 E}{L} & \frac{A_3 E}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ P \end{Bmatrix}$$

What happens if Q_J is not applied at a node?



$$f(x) = P\,\Delta(x - x_0) \qquad \int_{-\infty}^{\infty} \Delta(x - x_0) F(x)\,dx = F(x_0)$$

$$f_i^e = \int_{x_1}^{x_n} f(x)\, N_i(x)\,dx = \int_{x_1}^{x_n} P\,\Delta(x - x_0)\, N_i(x)\,dx = P\, N_i(x_0)$$



$$N_1 = \left(1 - \frac{x}{L}\right) \qquad N_2 = \frac{x}{L}$$

$$f_1 = P\left(1 - \frac{x_0}{L}\right) = \frac{P}{2} \quad , \quad f_2 = P\left(\frac{x_0}{L}\right) = \frac{P}{2} \qquad x_0 = \frac{L}{2}$$

$$-\frac{d}{dx}\left(a \frac{du}{dx}\right) + c\,u - f = 0 \qquad 0 < x < L$$

$$u(0) = u_0 \quad \& \quad \left(a \frac{du}{dx}\right)\Big|_{x=L} = Q_0$$

	u	a	c	f	Q
Heat transfer	Temp, T	Thermal cond. k	conv.	heat gen	Heat source, Q
Flow	Press, P	Mobility $\frac{K}{\mu}$	0	dist inflow	Point, P
Elasticity	Disp, u	stiffness AE	0	axial force	Point load

Ex

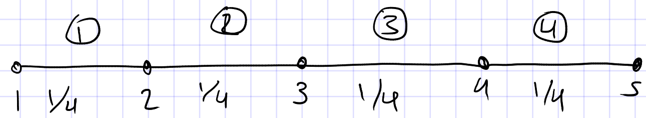
$$a = 1 \ , \quad c = 1 \ , \quad f = -x^2$$

$$-\frac{d^2u}{dx^2} - u + x^2 = 0 \qquad 0 < x < 1$$

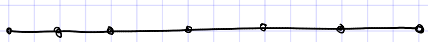
$$u(0) = 0 \qquad u(1) = 0$$

$$K_{IJ} = \int_0^1 \left(\frac{dN_I}{dx}\frac{dN_J}{dx} - N_I N_J\right) dx$$

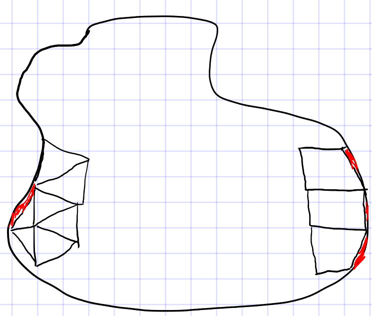
$$f_I = \int_0^1 (-x^2)\, N_J \, dx$$



2D scalar fields , u



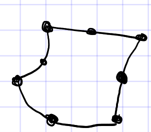
← no mesh error



Element
triangles
quads

elements ↑
size ↓

Degree of accuracy



→ no magic formula

General rules of thumb

1. Element should be able to reproduce fields on the order of the governing equations

2. #, shape, type element → accurate

3. The mesh density should cover areas of high gradients

4. Grade away gradually