$$c\phi \frac{\partial p}{\partial t} = \nabla\cdot\left(\frac{K}{\mu}\nabla p\right)$$

$$\int_\Omega w(\vec{x})\, c\phi \frac{\partial p}{\partial t}\, d\vec{x} - \int_\Omega w(\vec{x})\, \nabla\cdot\left(\frac{K}{\mu}\nabla p\right) d\vec{x} = 0$$

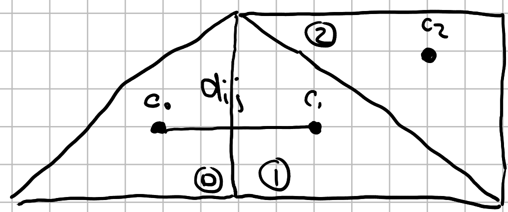
$$0 = \int_\Omega w(\vec{x})\, c\phi \frac{\partial p}{\partial t}\, d\vec{x} + \int_\Omega \frac{K}{\mu}\, \nabla w(\vec{x})\, \nabla p(\vec{x})\, d\vec{x} - \int_{\partial\Omega} w(\vec{x}) \frac{K}{\mu} \nabla p\cdot n\, ds$$

$$w(\vec{x}) = 1$$

$$0 = \int_\Omega c\phi \frac{\partial p}{\partial t}\, d\vec{x} - \int_{\partial\Omega} \frac{K}{\mu} \nabla p\cdot\vec{n}\, dA$$

Assume $p_i = \frac{1}{V_i}\int_{\Omega_i} p(x)\, dV$

$$V_i\, c\phi\, \frac{p^{n+1} - p^n}{\Delta t} - \sum_{f\in\partial\Omega_i} \overbrace{\frac{K}{\mu}\left(\nabla p^{n+1}\cdot\hat{n}\right)\cdot A_f}^{q_f} = 0$$

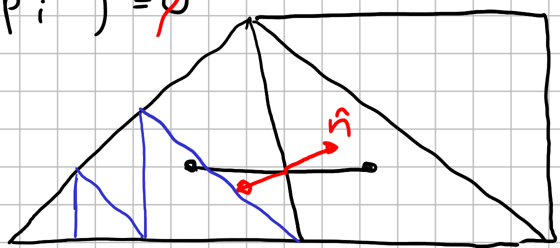


"Two point flux approximation"

$$(\nabla p\cdot n)_f \approx \frac{p_j - p_i}{d_{ij}}$$

$$d_{ij} = \|\vec{c}_j - \vec{c}_i\|$$

$$\underbrace{\frac{V_i\, c\phi}{B_w\, \Delta t}}_{[B]}\left(p_i^{n+1} - p_i^n\right) - \sum_{f\in\Omega_i} \overbrace{\underbrace{\frac{K A_f}{\mu\, d_{ij}\, B_w}}_{[T]}}^{g}\left(p_j^{n+1} - p_i^{n+1}\right) = \cancel{0}\ r$$



$$\vec{r}(\vec{p}) = 0$$

$$\vec{r}(\vec{p}) = 0$$

Compute a Jacobian

$$J = \frac{\partial \vec{r}}{\partial \vec{p}} = \frac{\partial r_l}{\partial p_m} = J_{lm}$$

Solve

$$[J]\, \Delta\vec{p} = -\vec{r}$$

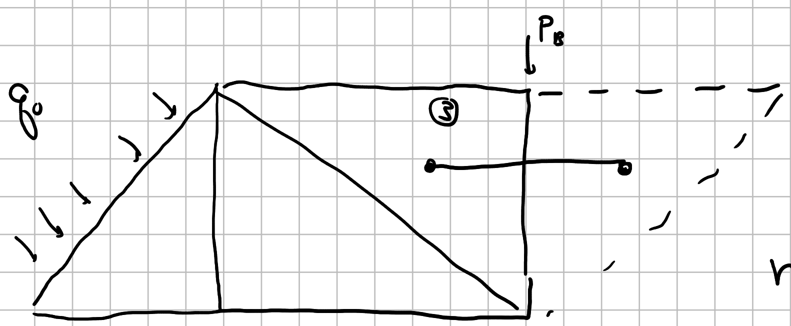
Iterate

$$^{k+1}p = {}^{k}p + \Delta p$$

$$\vec{r}(\vec{p}) = \begin{bmatrix} p_0 + p_1 \\ p_0 p_1 \end{bmatrix} \Rightarrow \frac{\partial \vec{r}}{\partial \vec{p}} = \left.\begin{bmatrix} 1 & p_1 \\ 1 & p_0 \end{bmatrix}\right|_{\vec{p}=(2,3)}$$

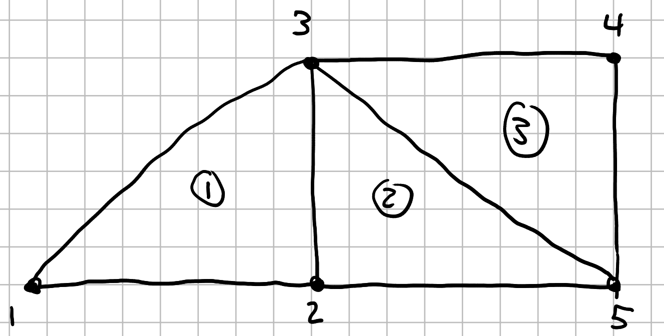
$$= \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

Let's use JAX to do the same.



$$\vec{Q}_3 = 2 T_0 P_B$$

$$n = \begin{bmatrix} 0, & 0 \\ 1, & 0 \\ 1, & 1 \\ 2, & 1 \\ 2, & 0 \end{bmatrix}$$



$$C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$$